The Mathematical Theory of Communication

International Morse Code

1. The length of a dot is one unit.
2. A dash is three units.
3. The space between parts of the same letter is one unit.
4. The space between letters is three units.
5. The space between words is seven units.

A	• —	U	• • —
B	— • • •	V	• • • —
C	— • — •	W	• — —
D	— • •	X	— • • —
E	•	Y	— • — —
F	• • — •	Z	— — • •
G	— — •		
H	• • • •		
I	• •		
J	• — — —		
K	— • —	1	• — — — —
L	• — • •	2	• • — — —
M	— —	3	• • • — —
N	— •	4	• • • • —
O	— — —	5	• • • • •
P	• — — •	6	— • • • •
Q	— — • —	7	— — • • •
R	• — •	8	— — — • •
S	• • •	9	— — — — •
T	—	0	— — — — —

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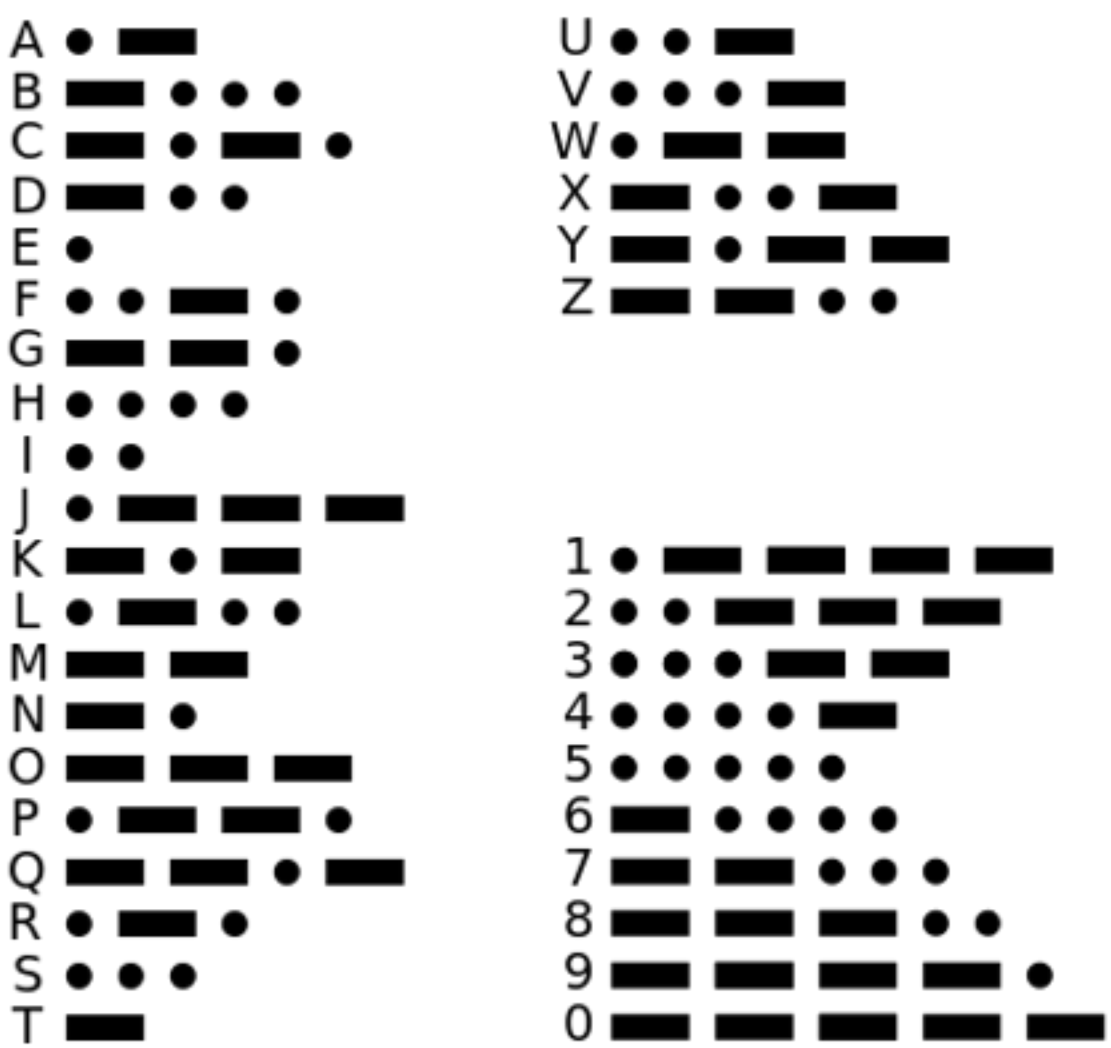
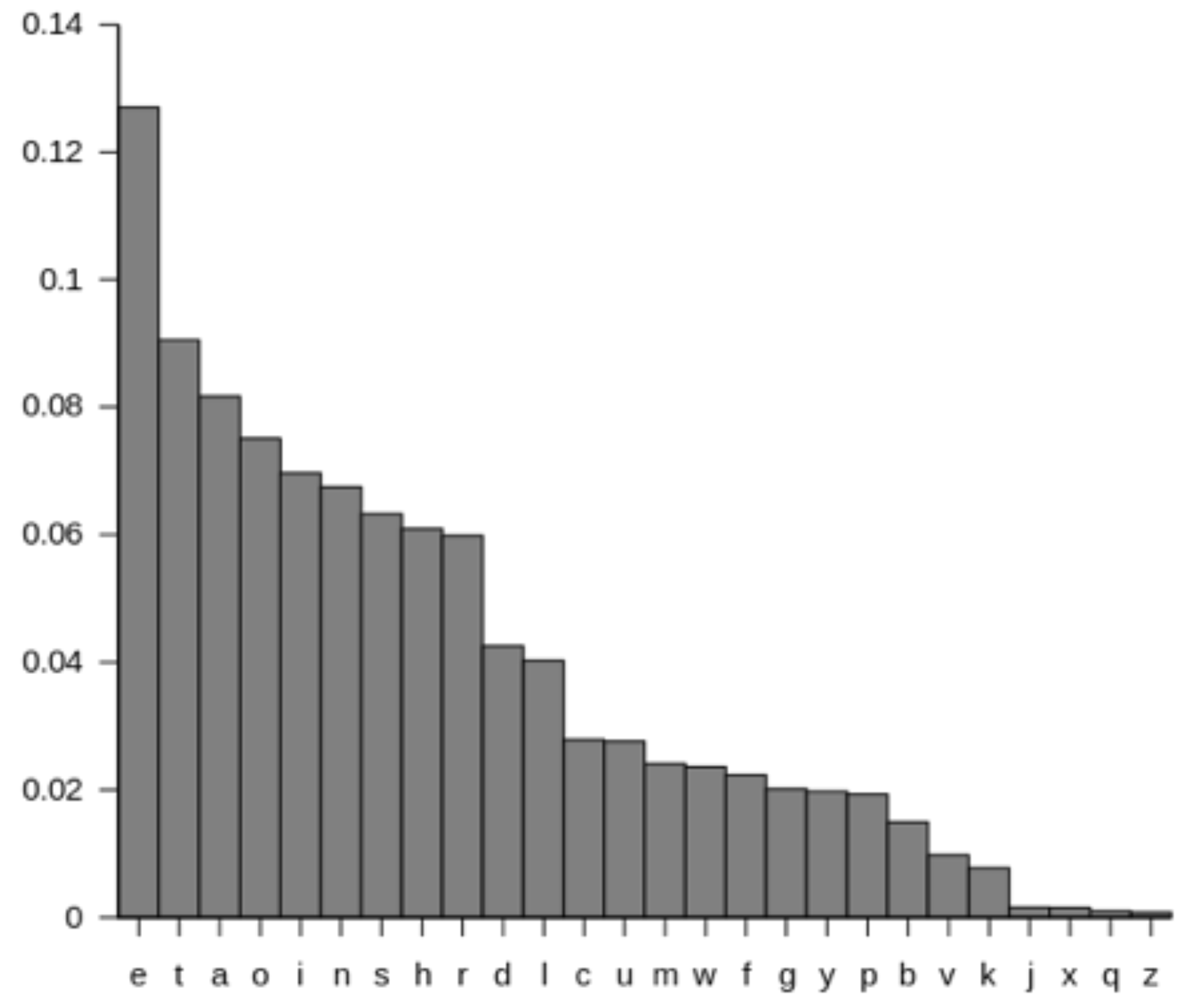




Harry Nyquist

- *Certain Factors Affecting Telegraph Speed*. Bell Labs Technical Journal. 1924

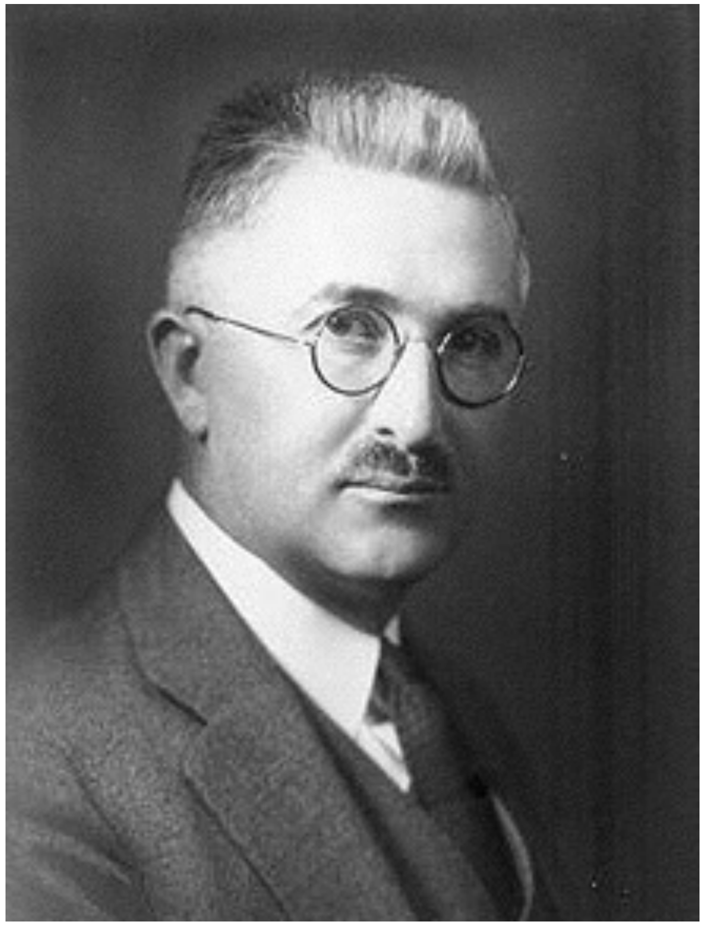
$$W = K \log m$$

Where W is the speed of transmission of intelligence,
m is the number of current values,
and, K is a constant.

Ralph Hartley

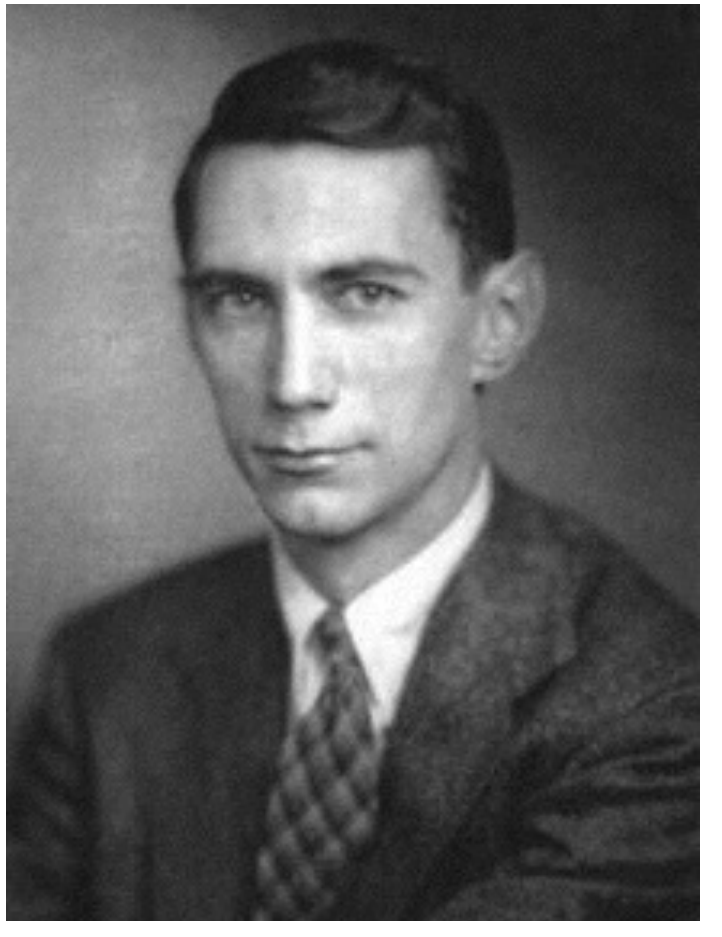
- *Transmission of Information*. Bell Labs Technical Journal. 1928

$$H = n \log s$$
$$= \log s^n.$$



Claude Shannon

- *A Symbolic Analysis of Relay and Switching Circuits*. Master's Thesis. MIT. 1937



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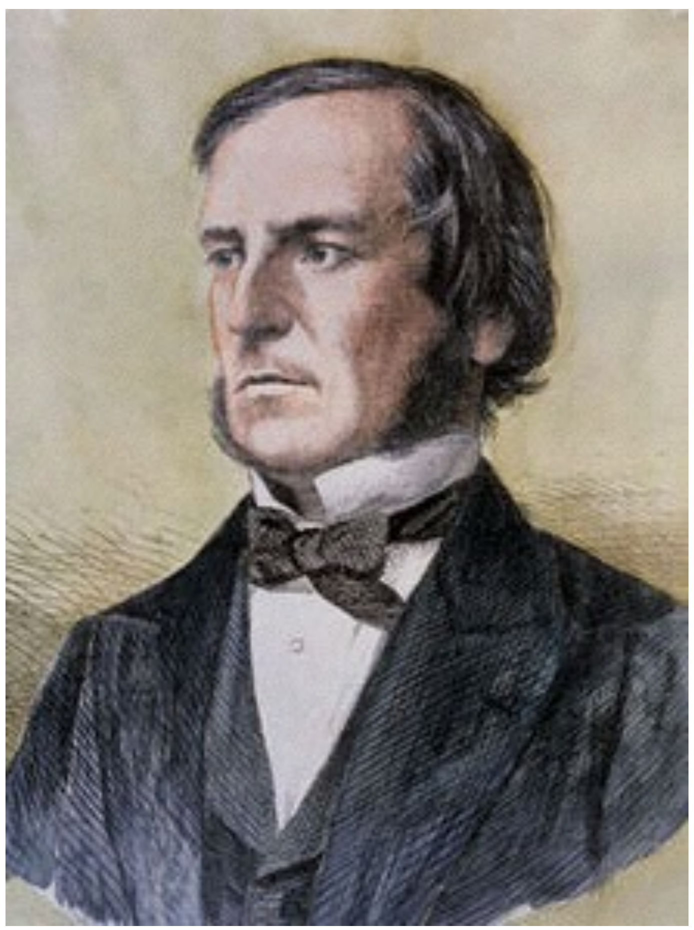
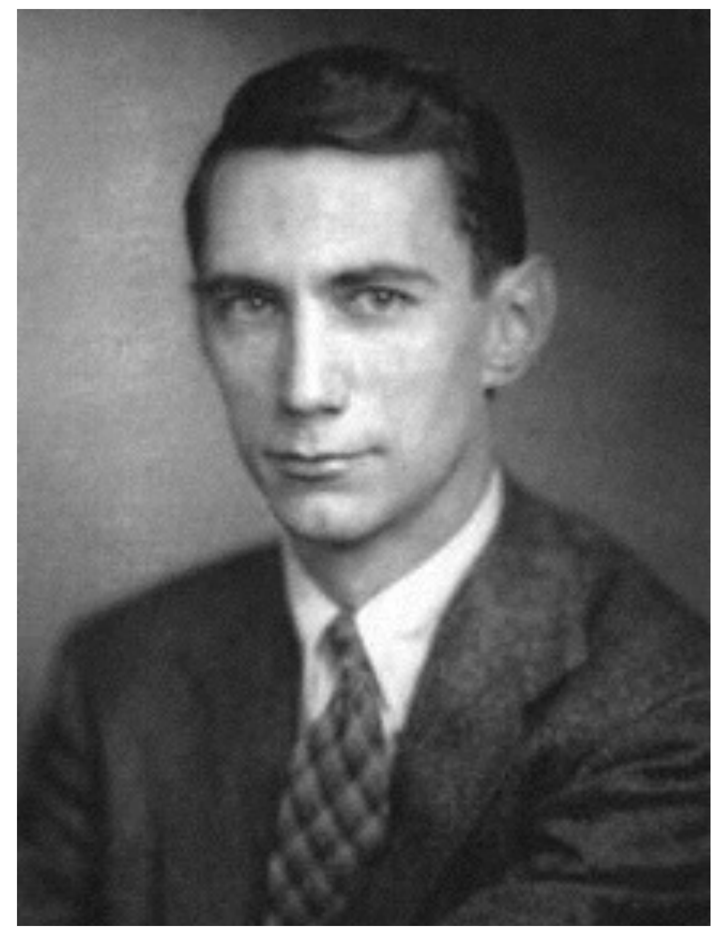
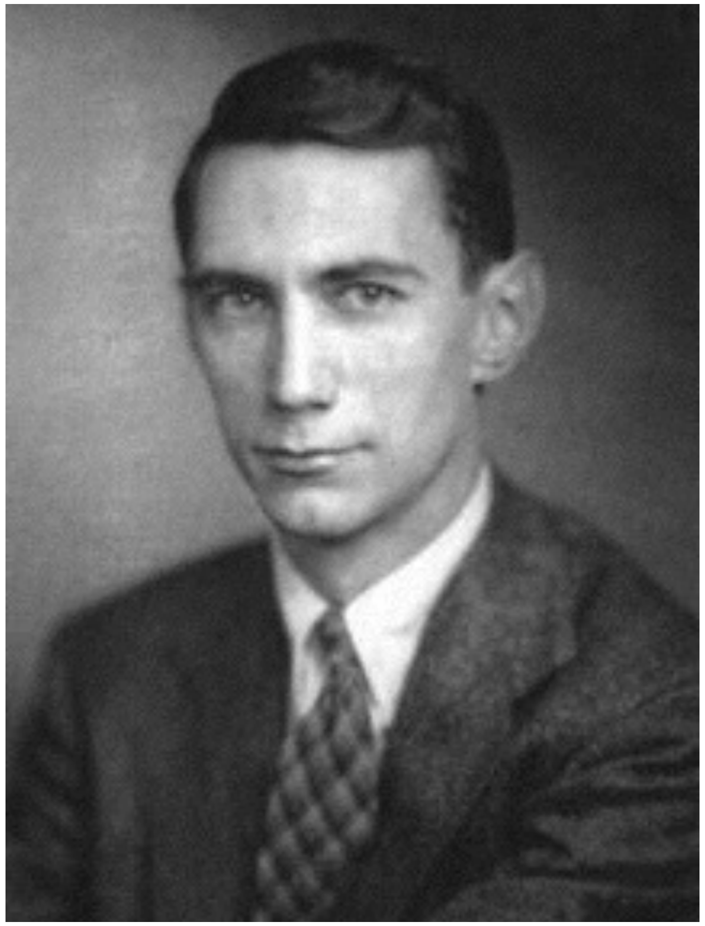




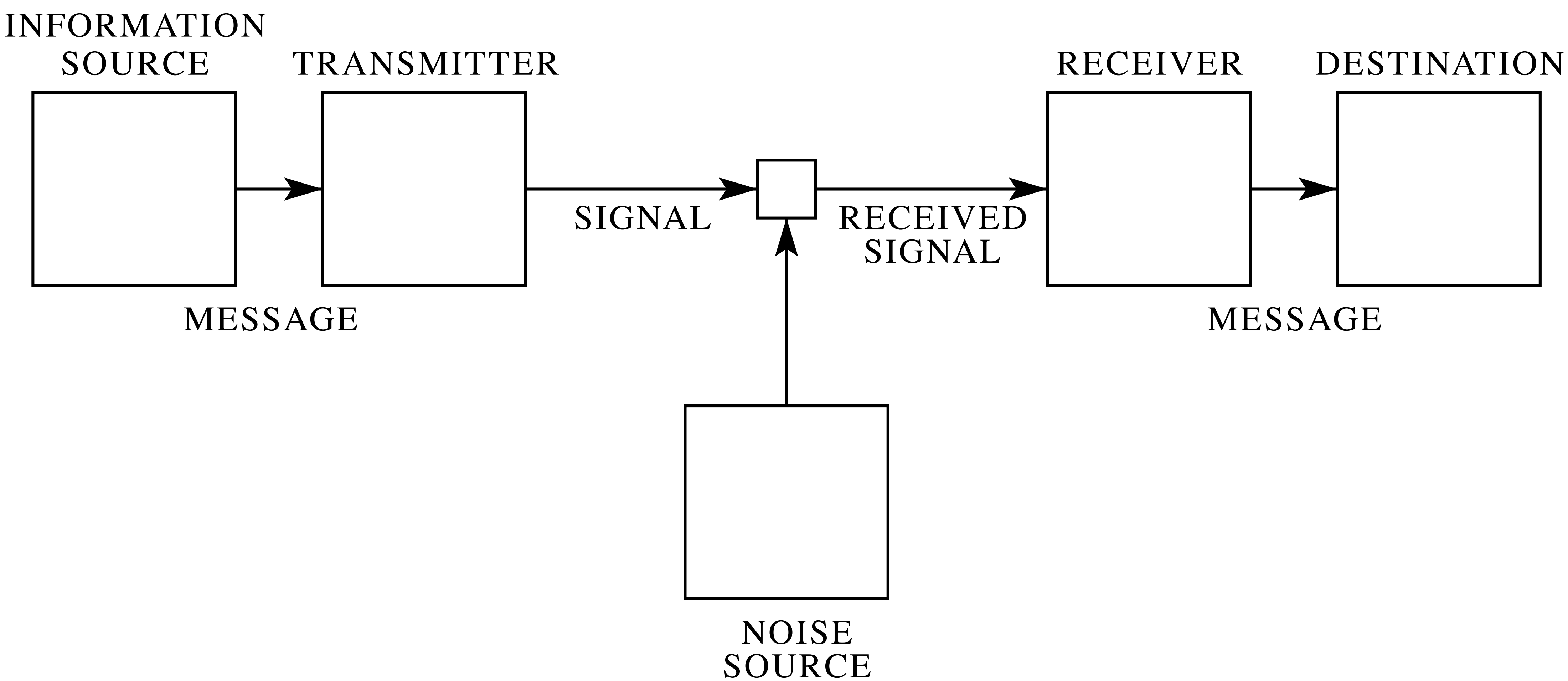
Claude Shannon

- *A Mathematical Theory of Communication*. Bell Labs Technical Journal. 1948

$$H = -\sum p_i \log p_i$$



The Noisy Channel Model



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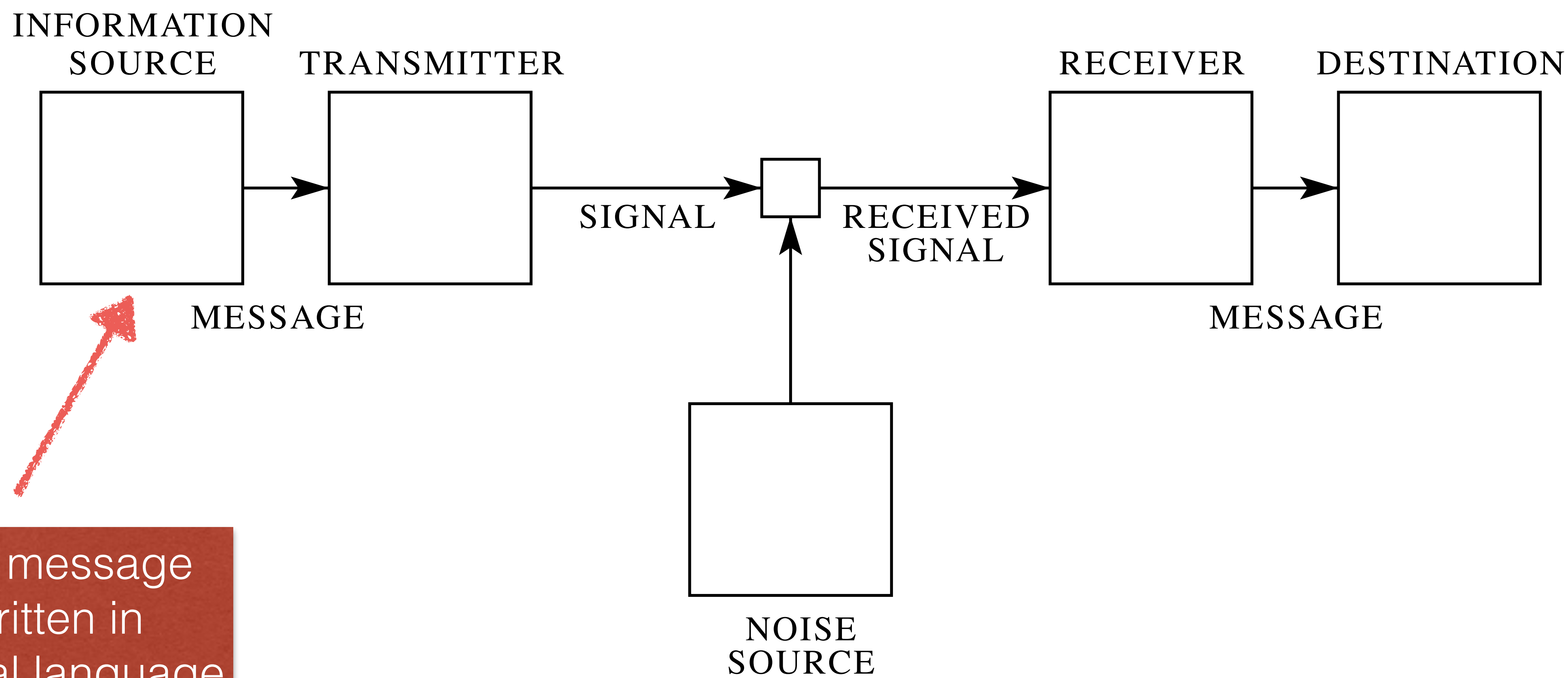
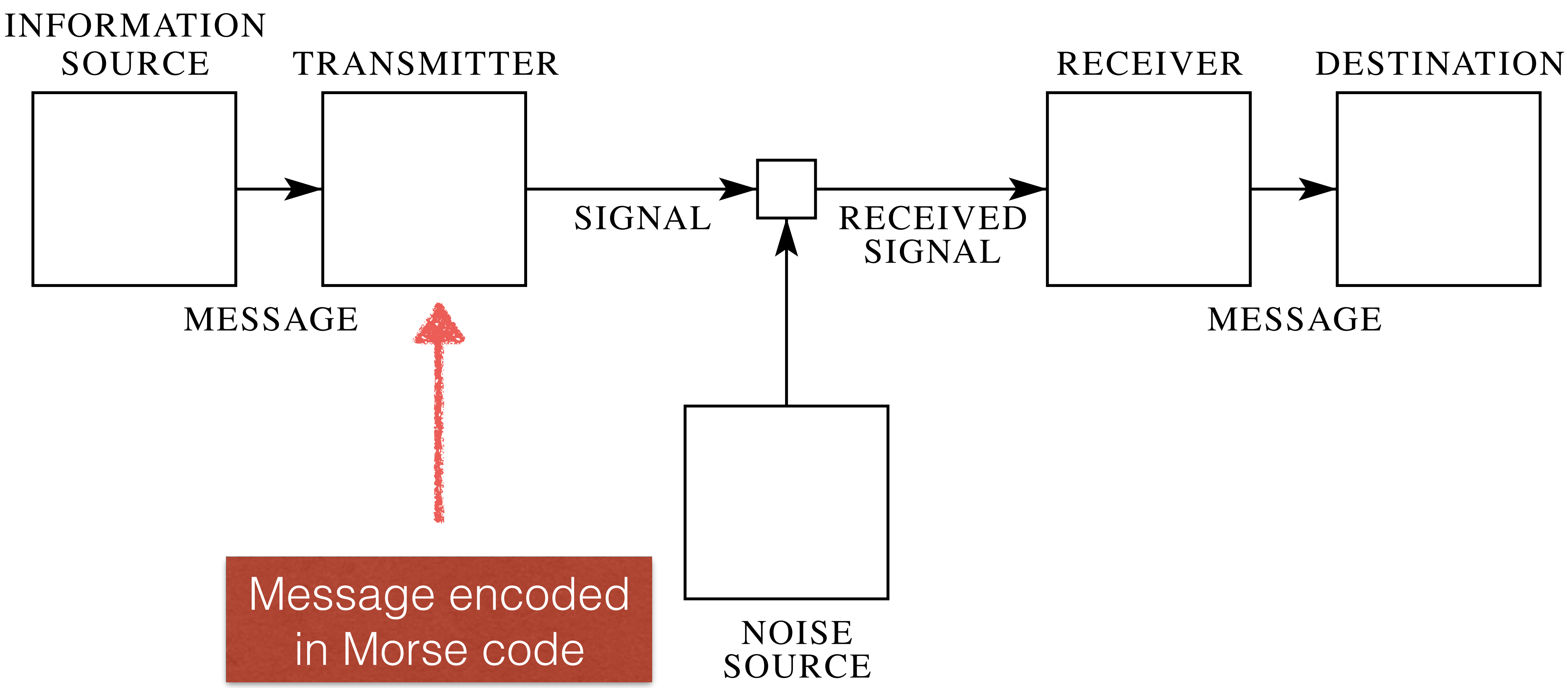


Text message written in natural language

The Noisy Channel Model



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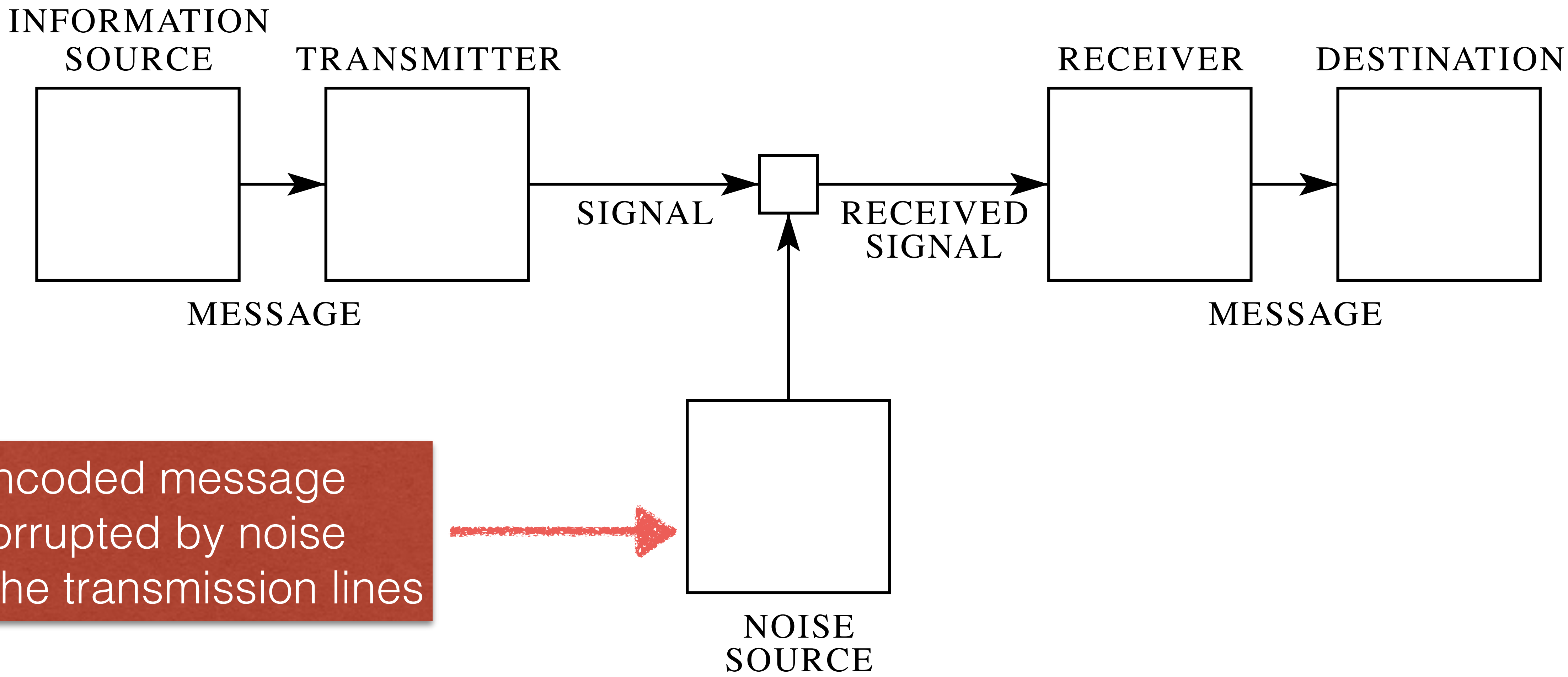
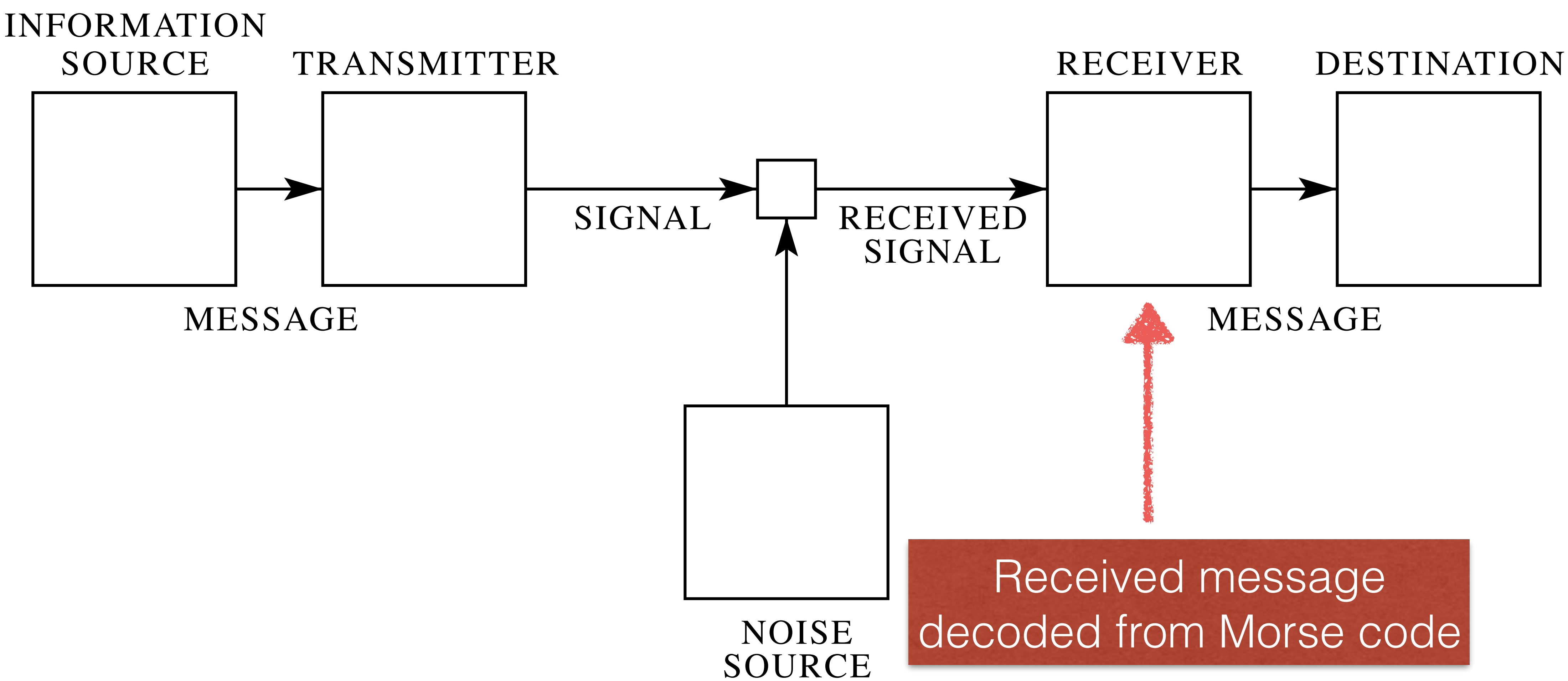


Encoded message corrupted by noise from the transmission lines

The Noisy Channel Model



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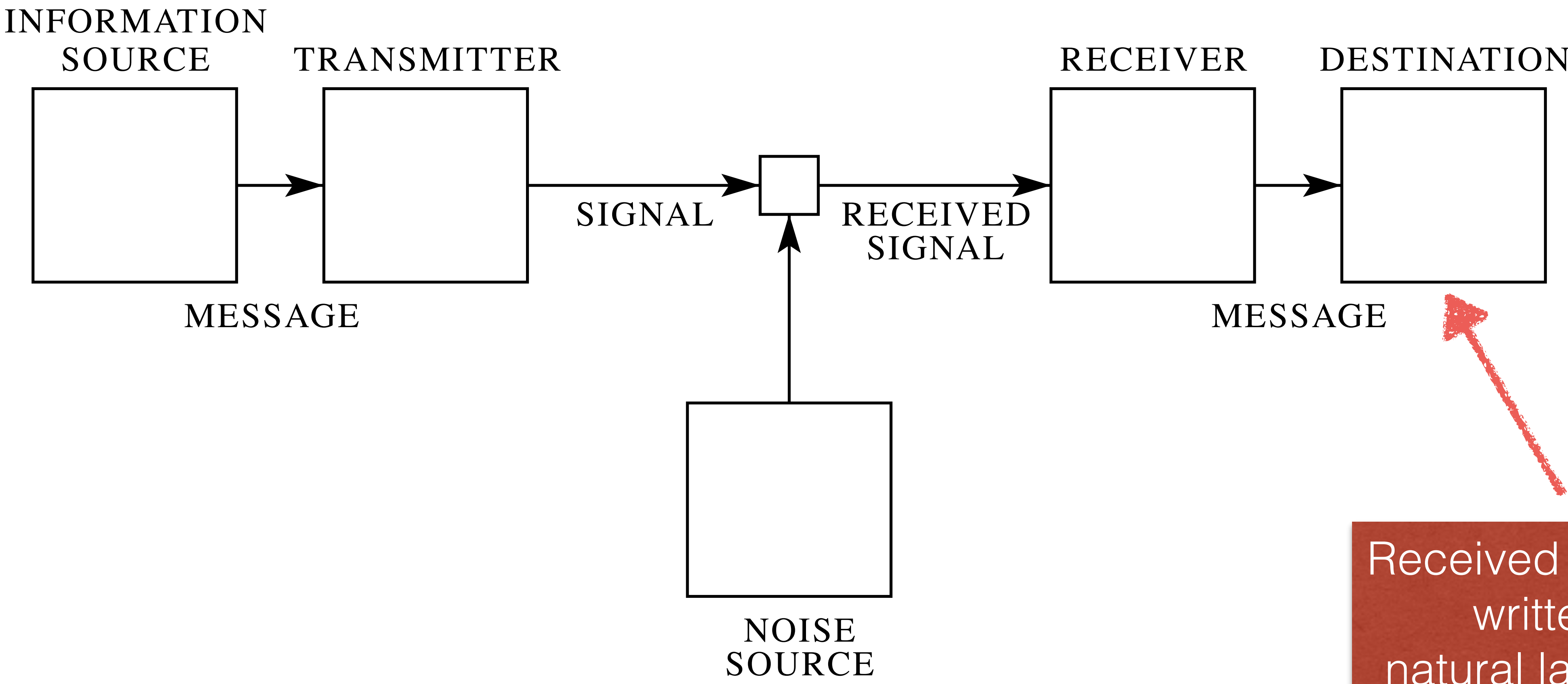

Received message written in natural language. May contain errors.

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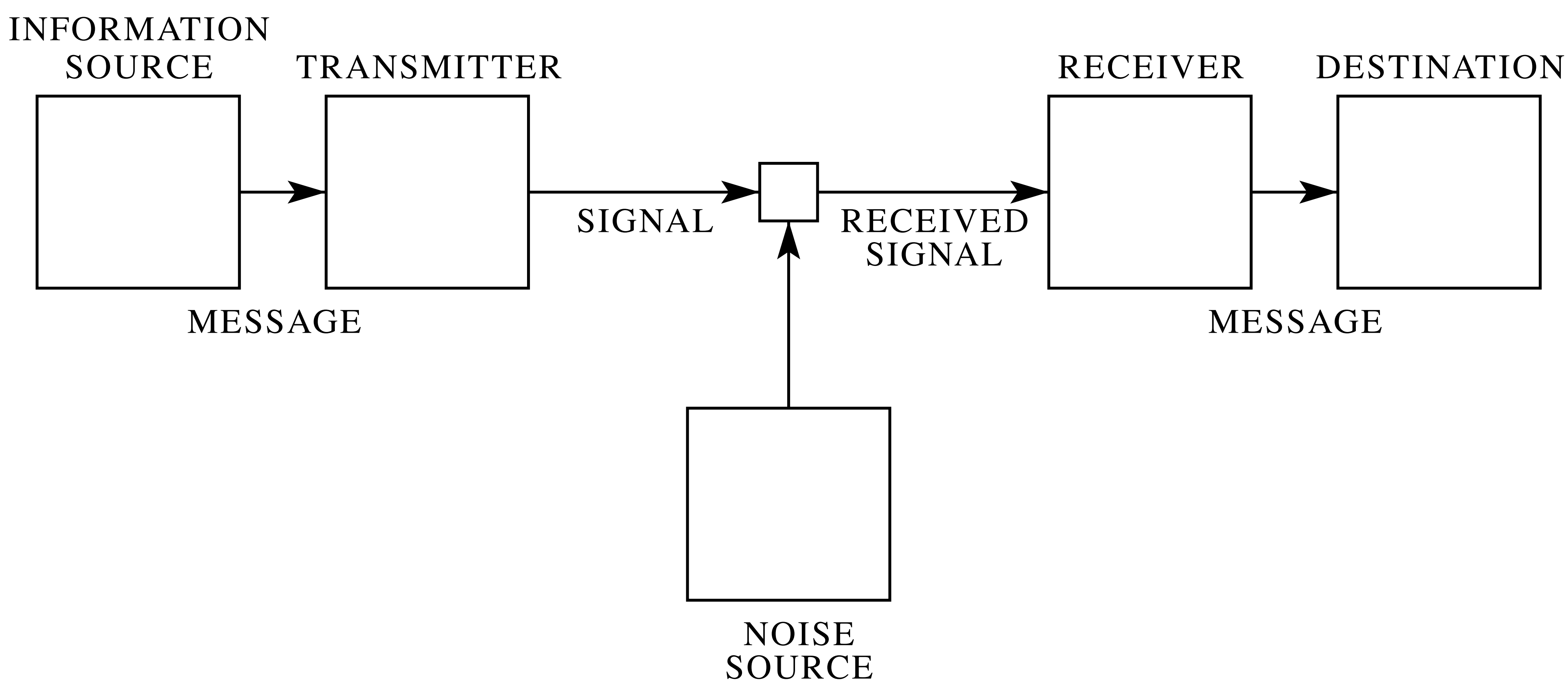

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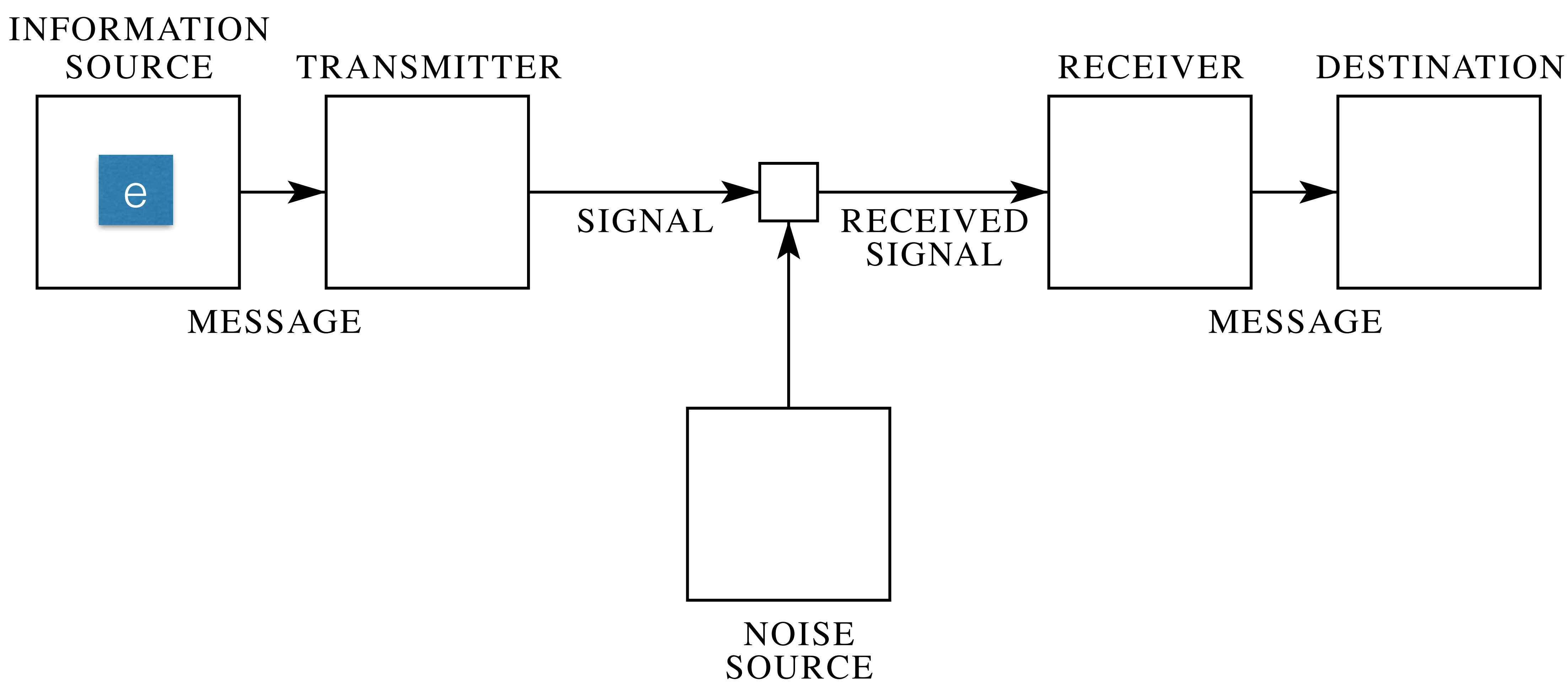

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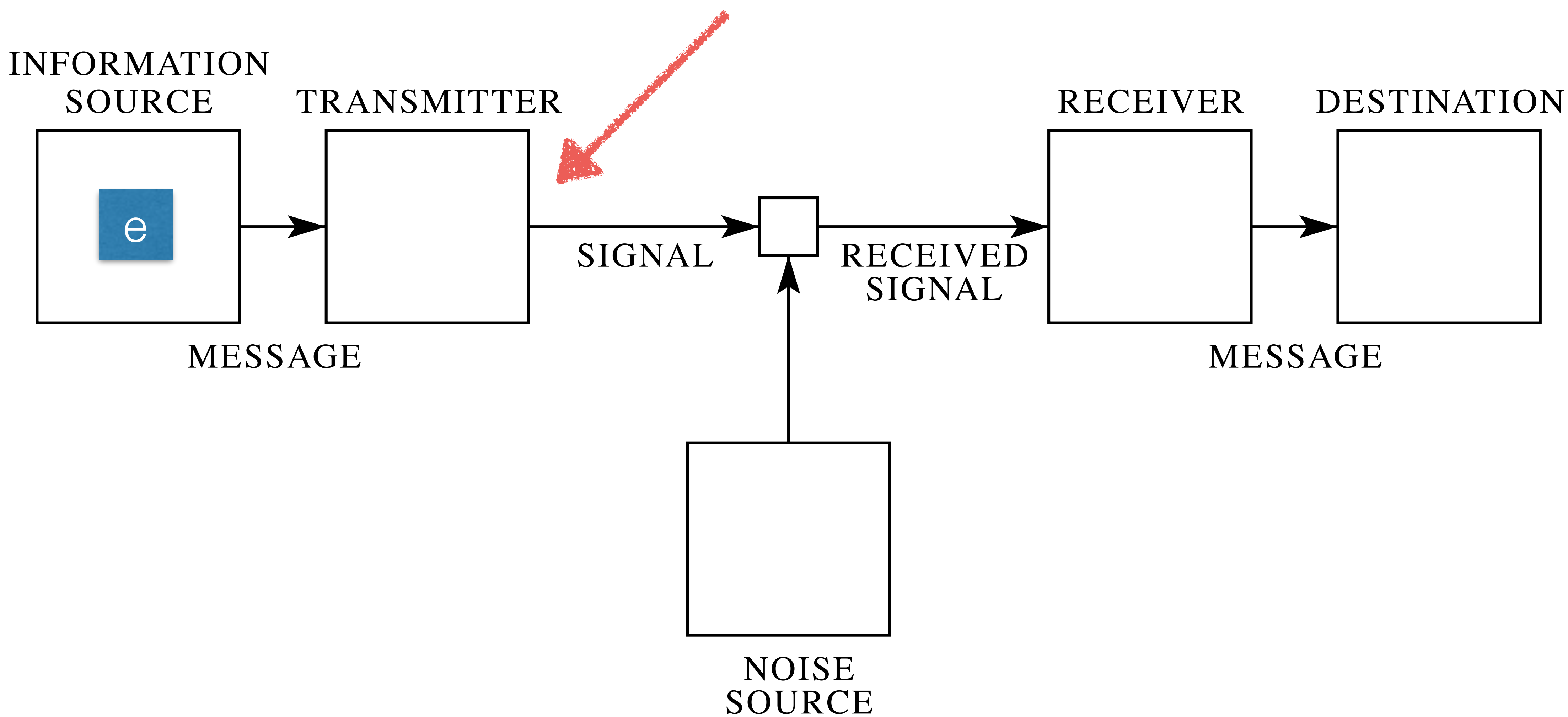

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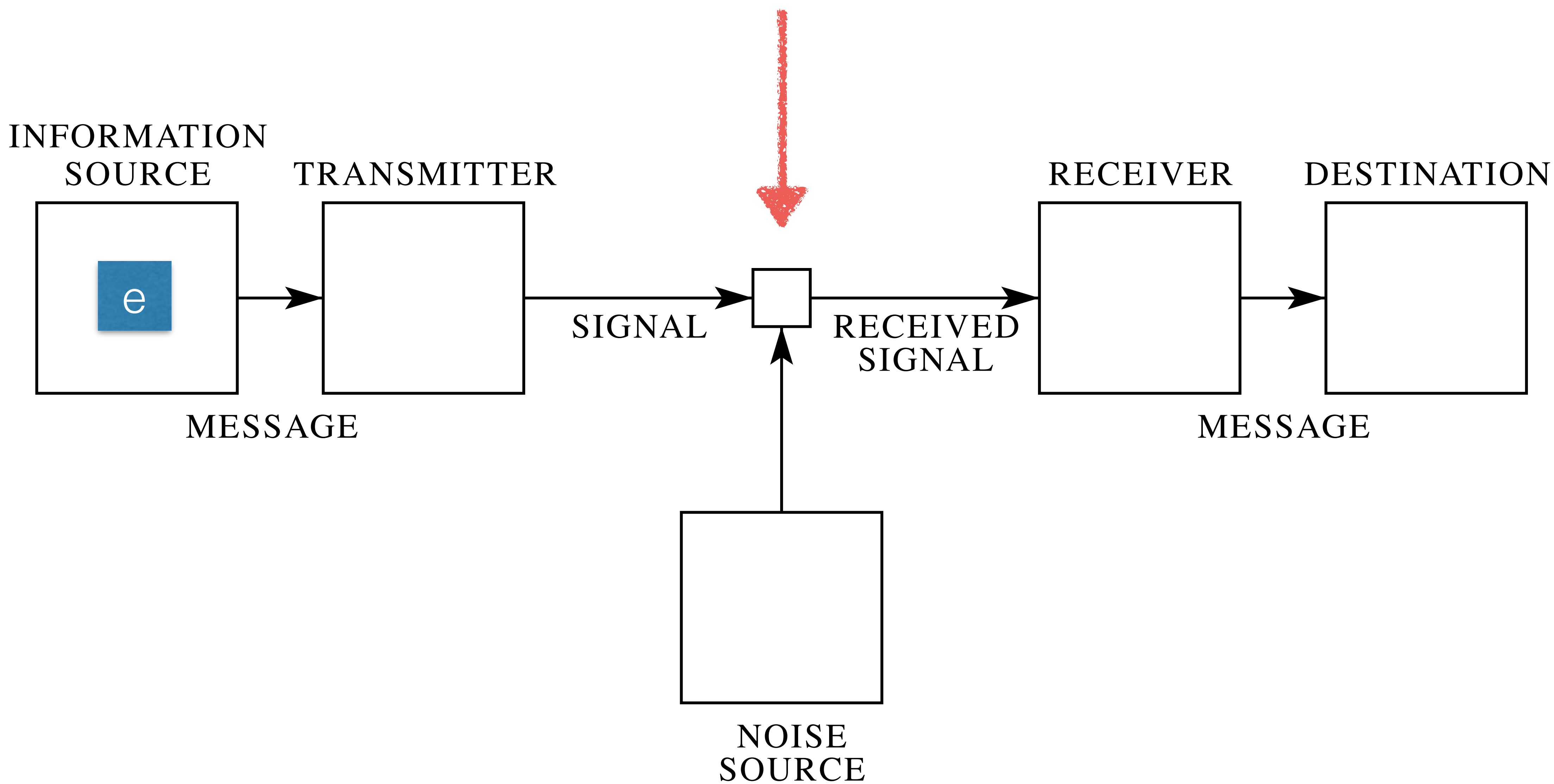

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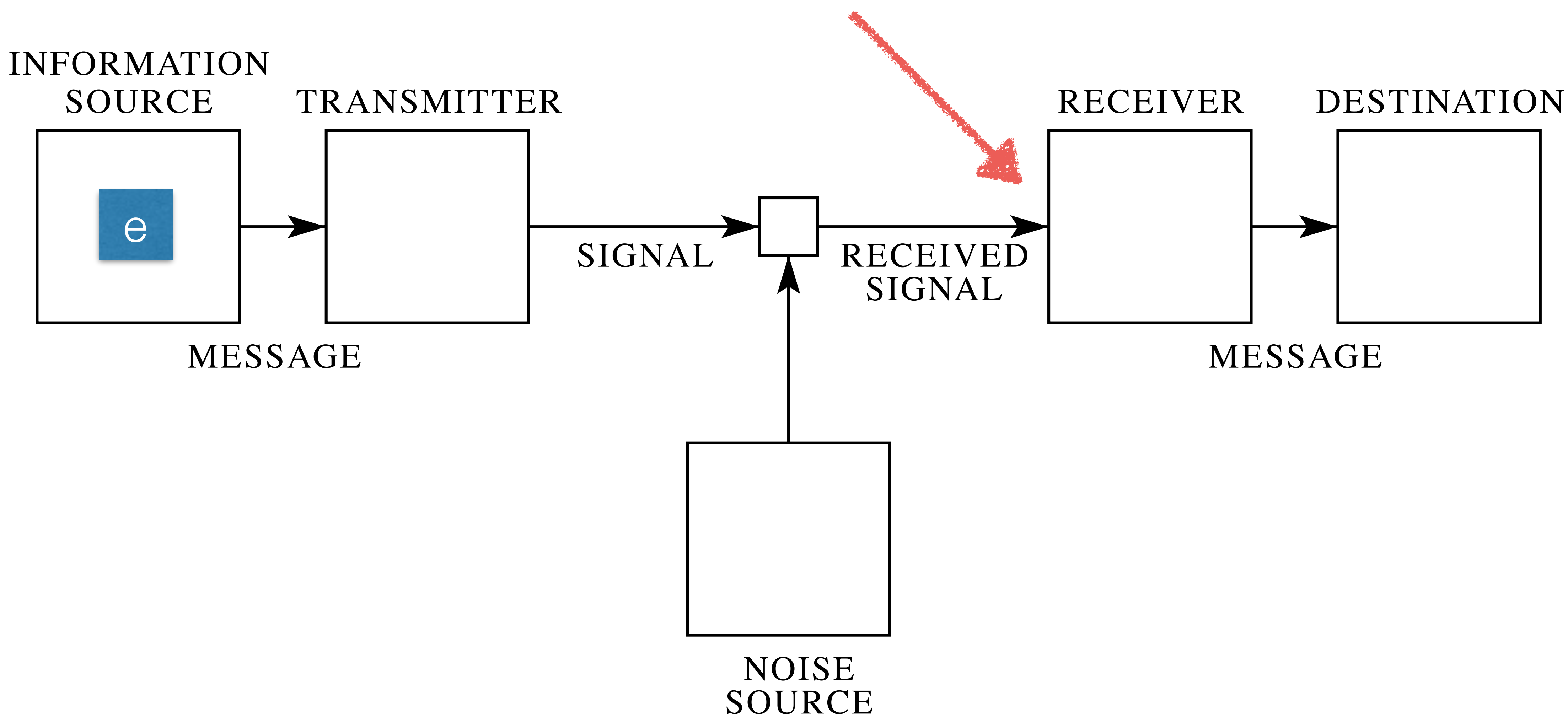

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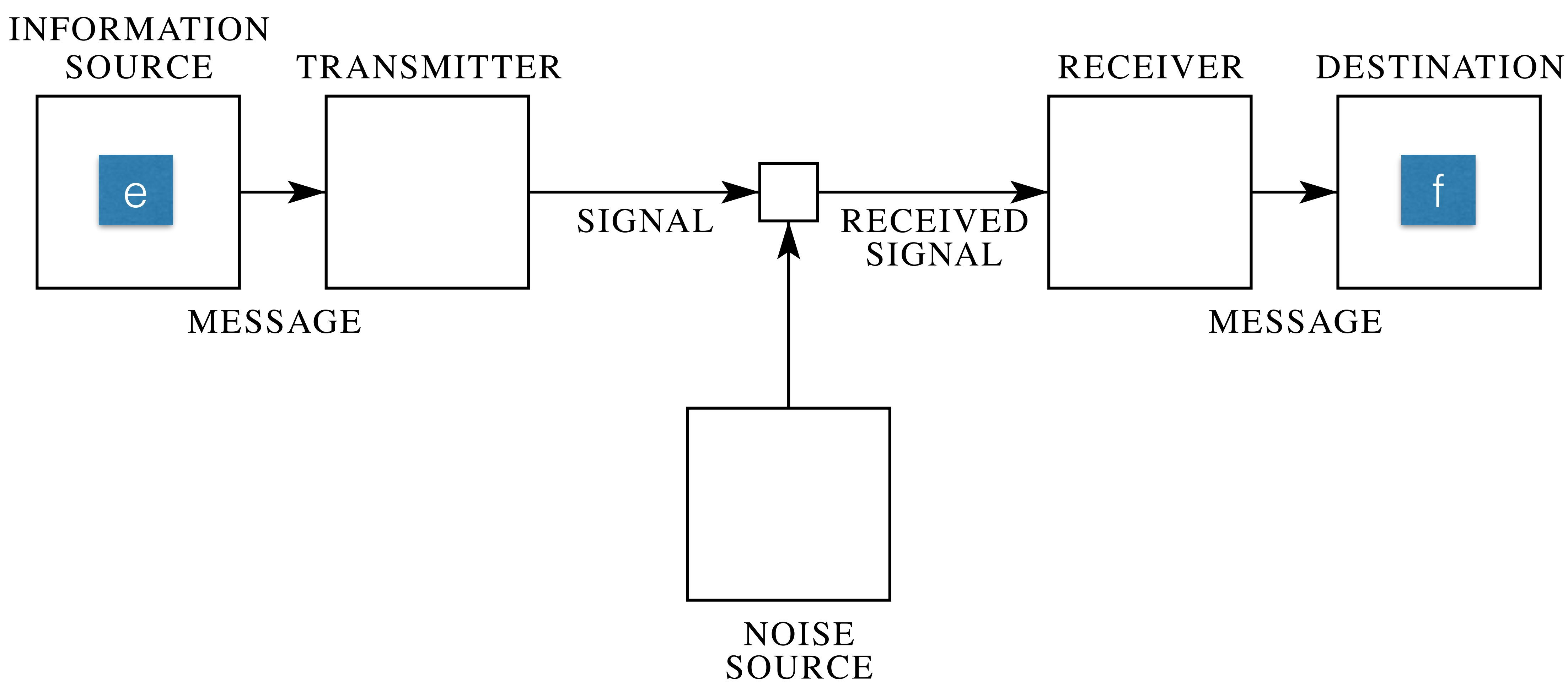

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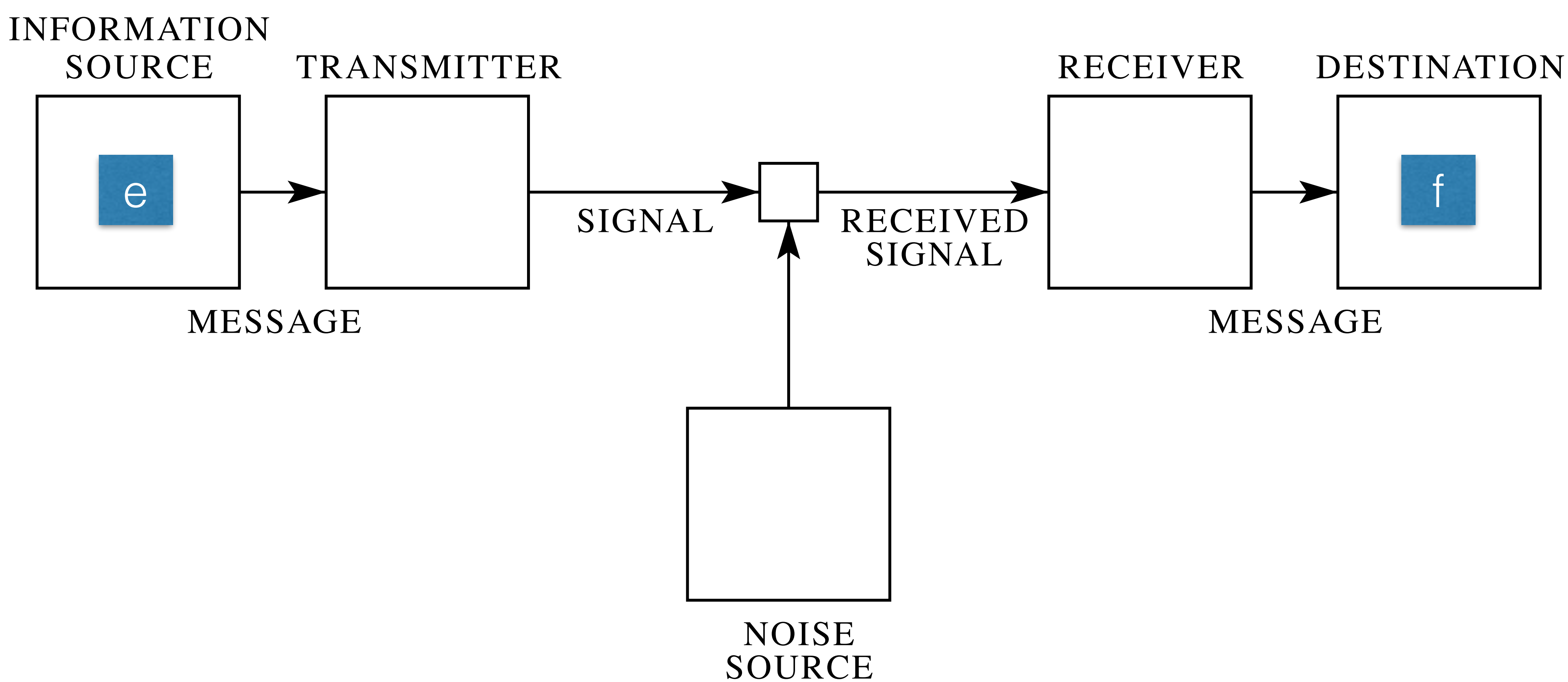
$$\hat{\mathbf{e}} = \arg\max_{\mathbf{e}} p(\mathbf{e}|\mathbf{f})$$



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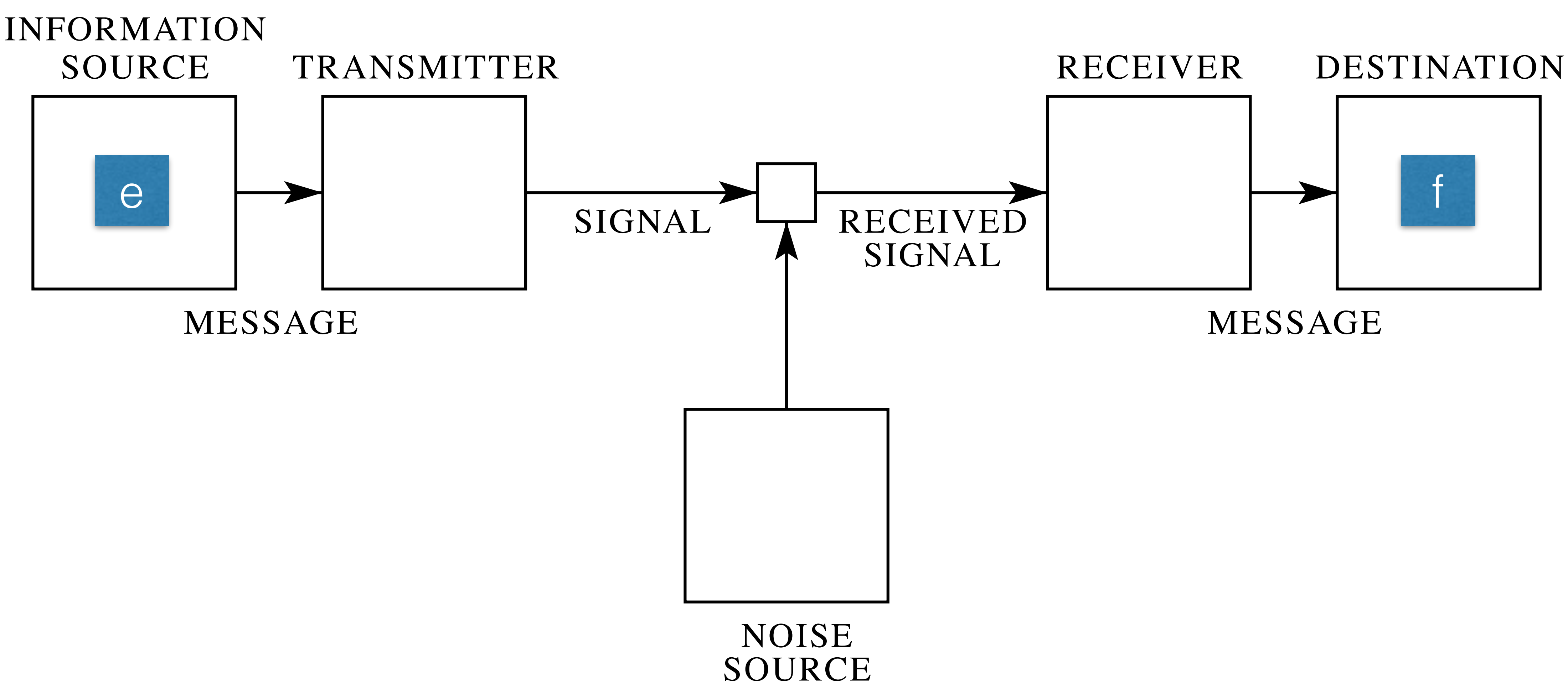

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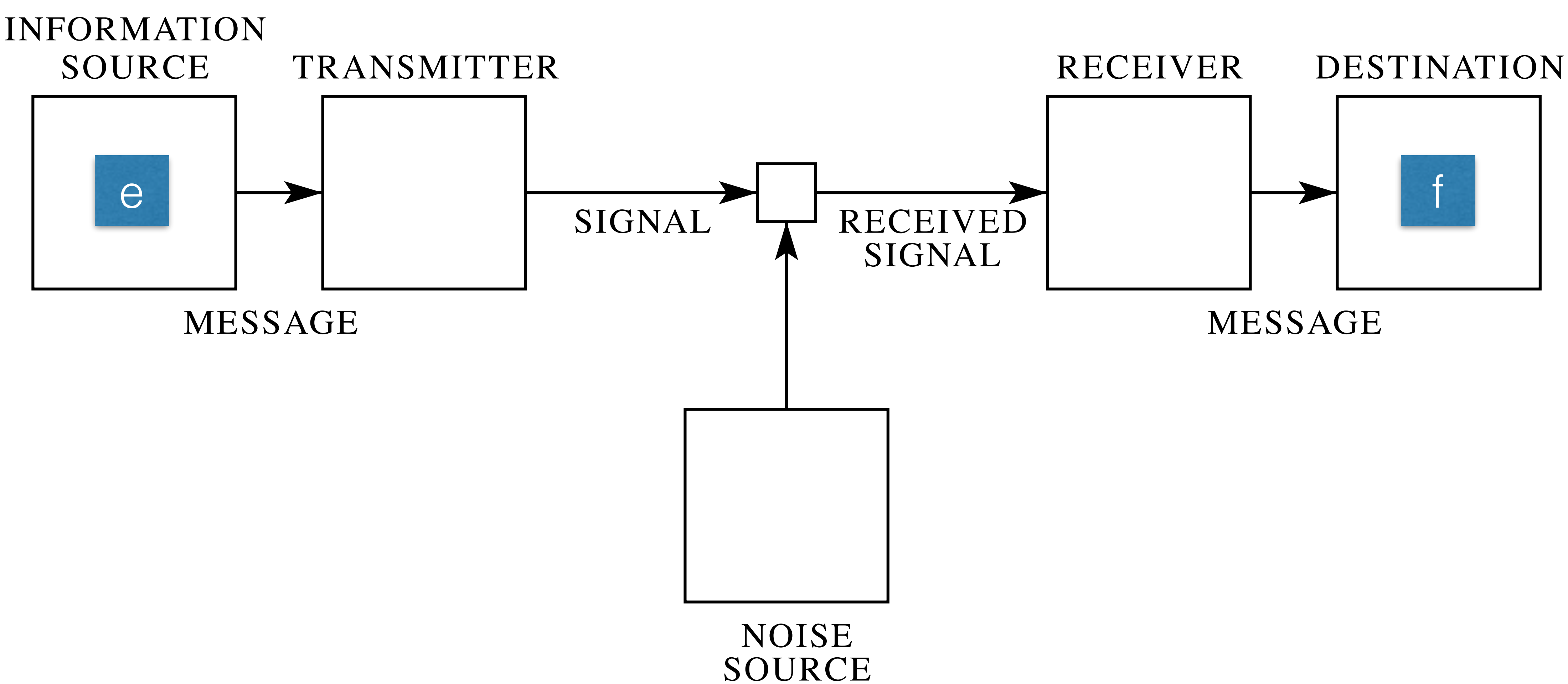

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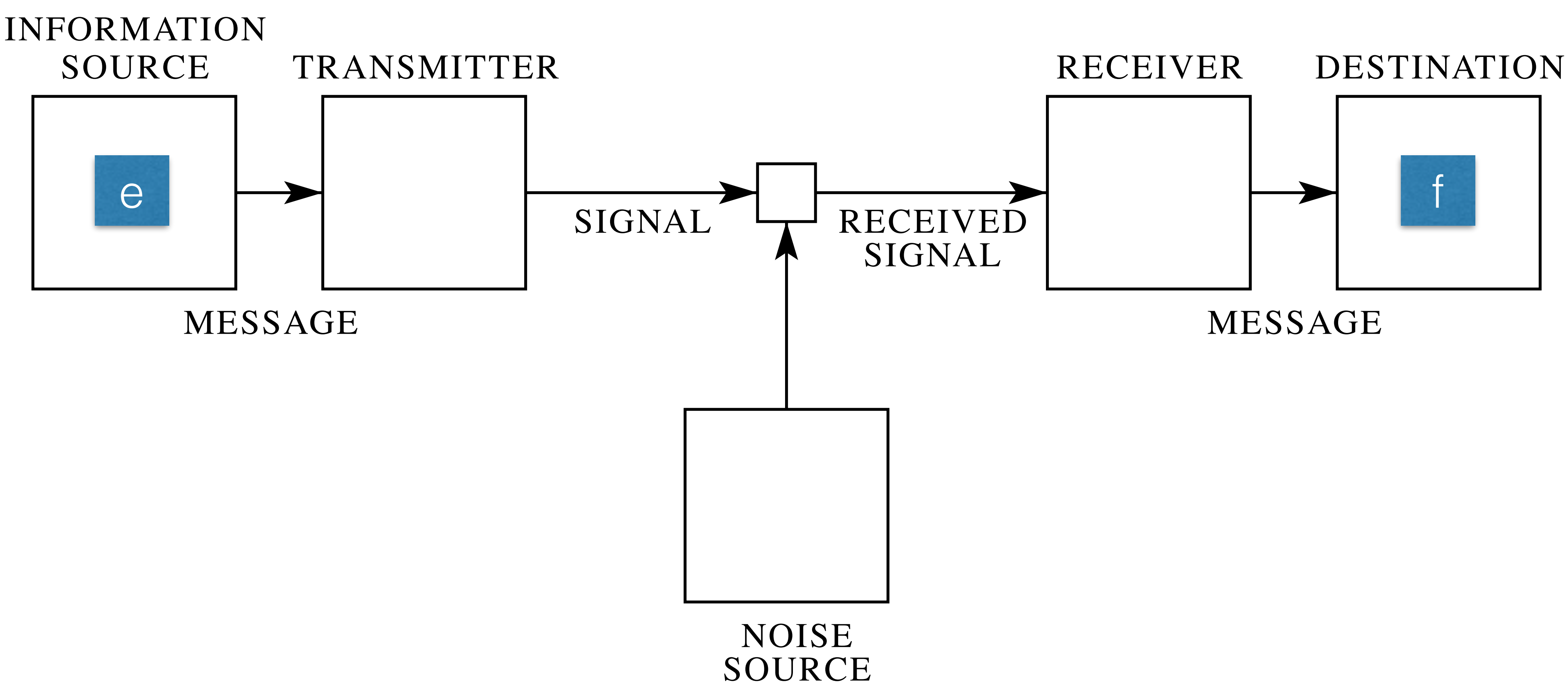

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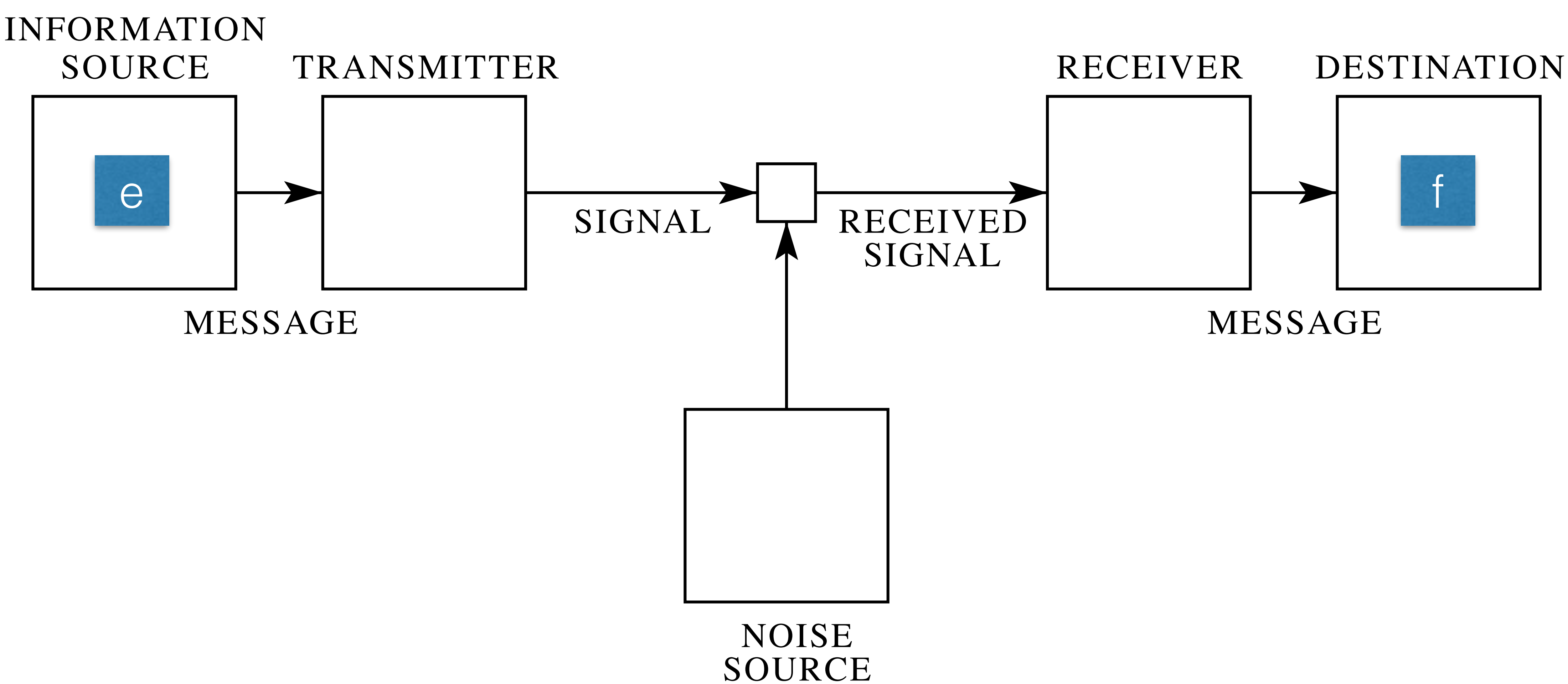

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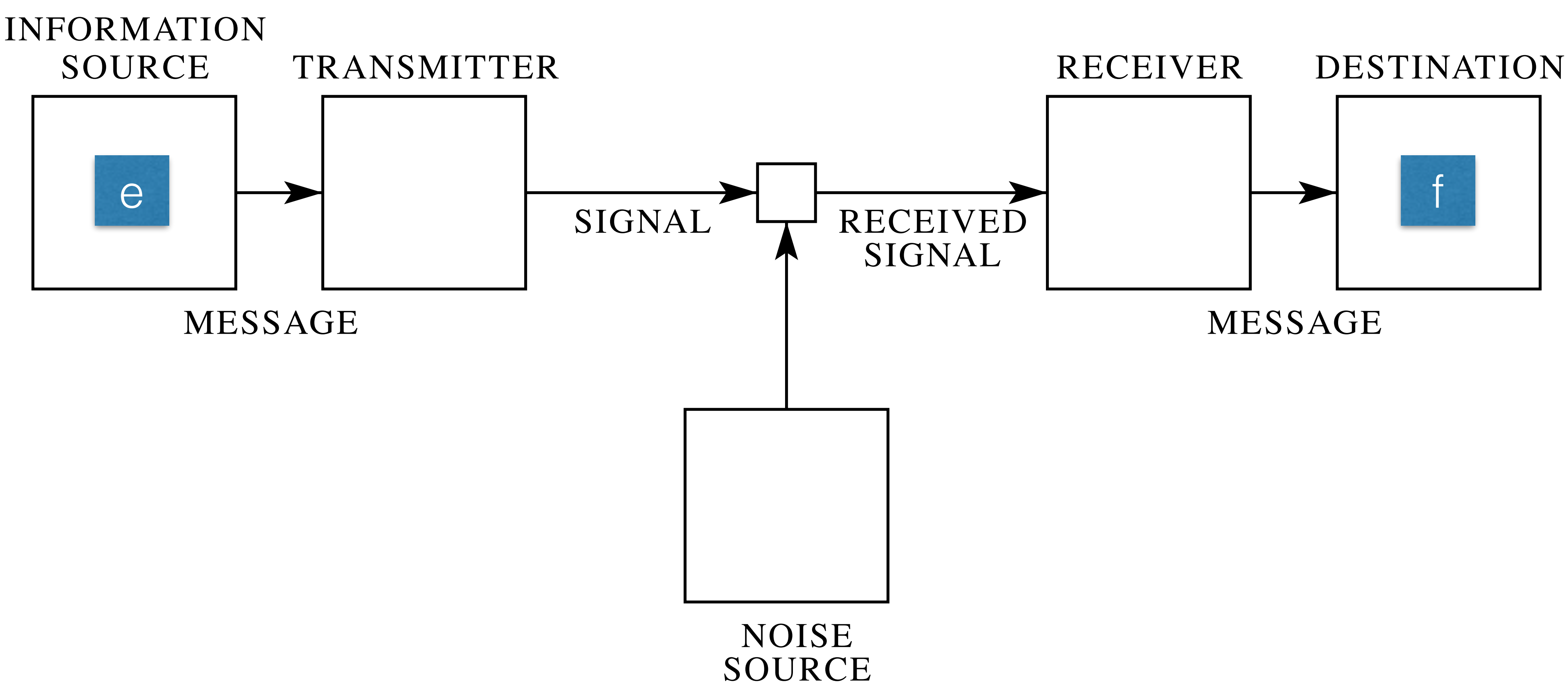

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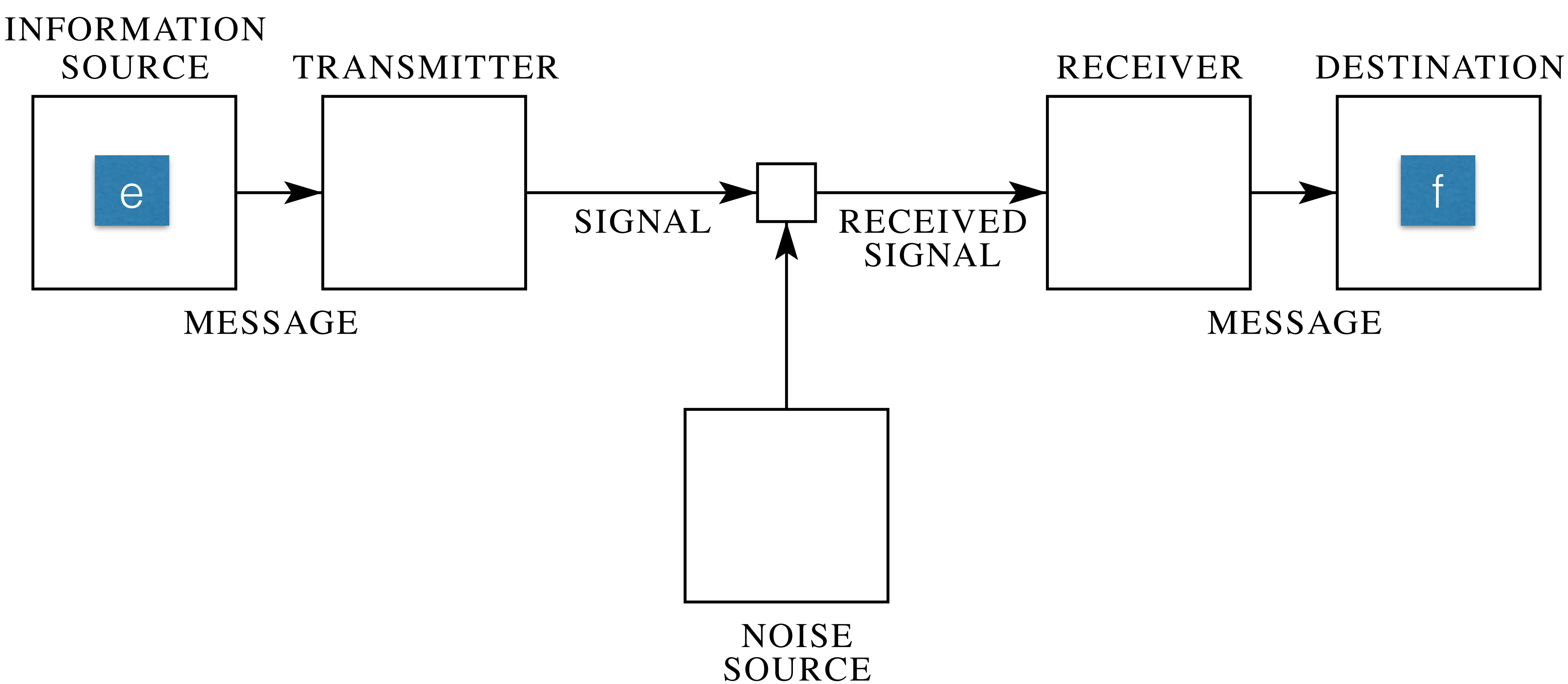

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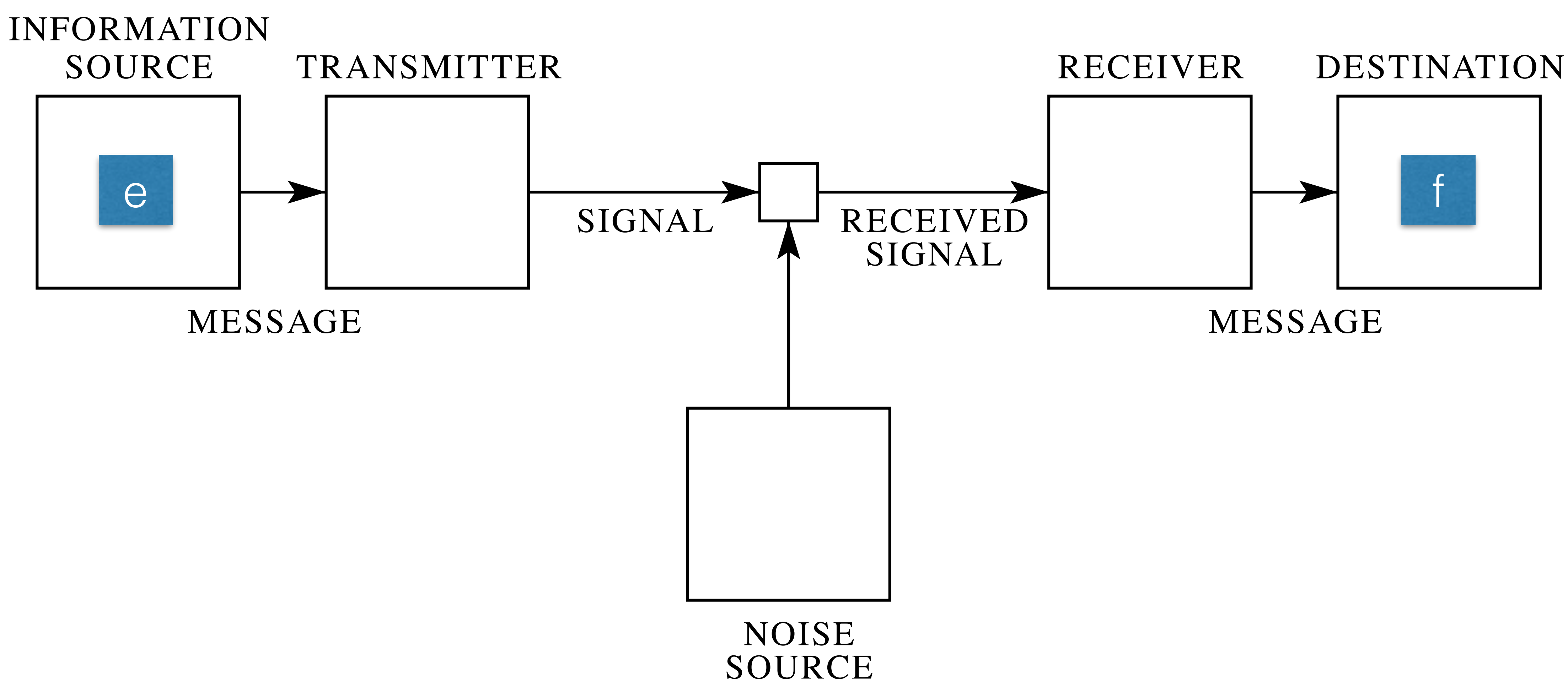

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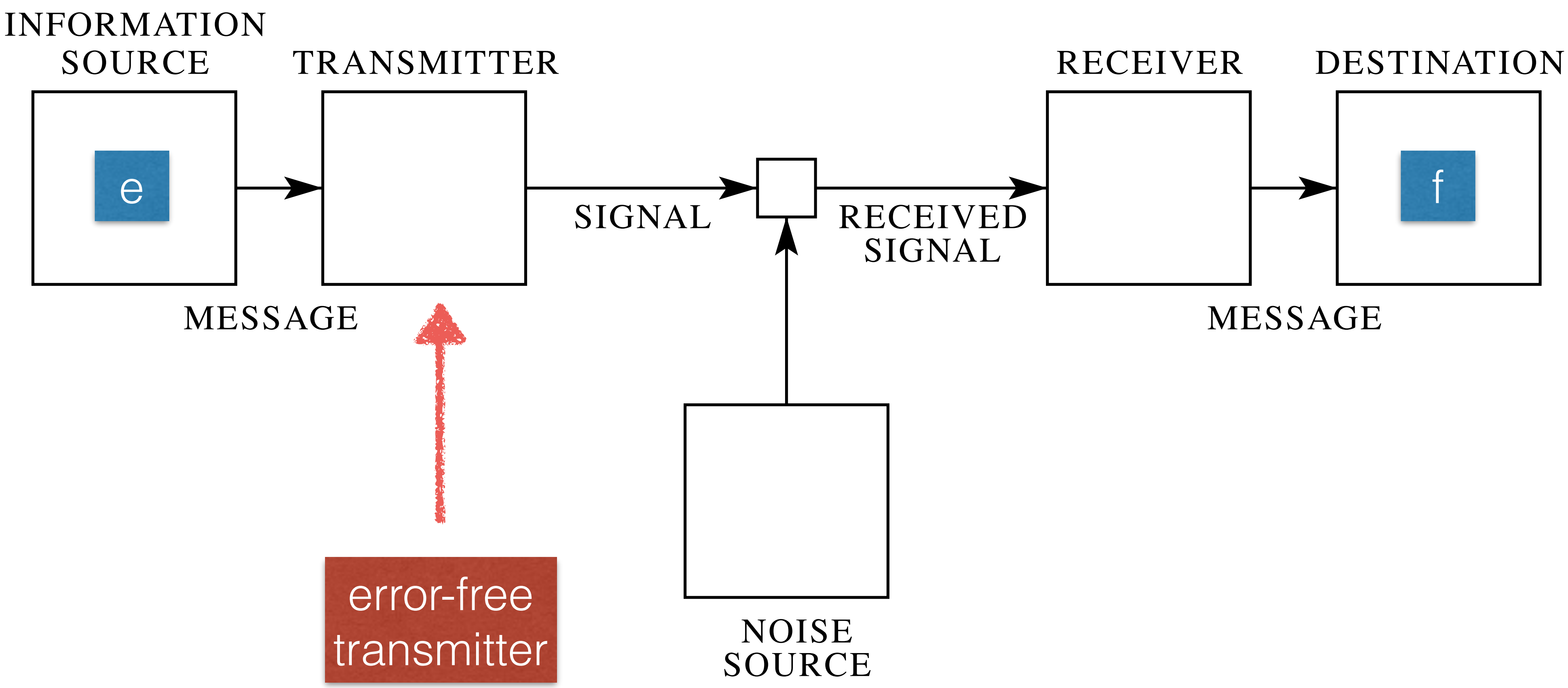

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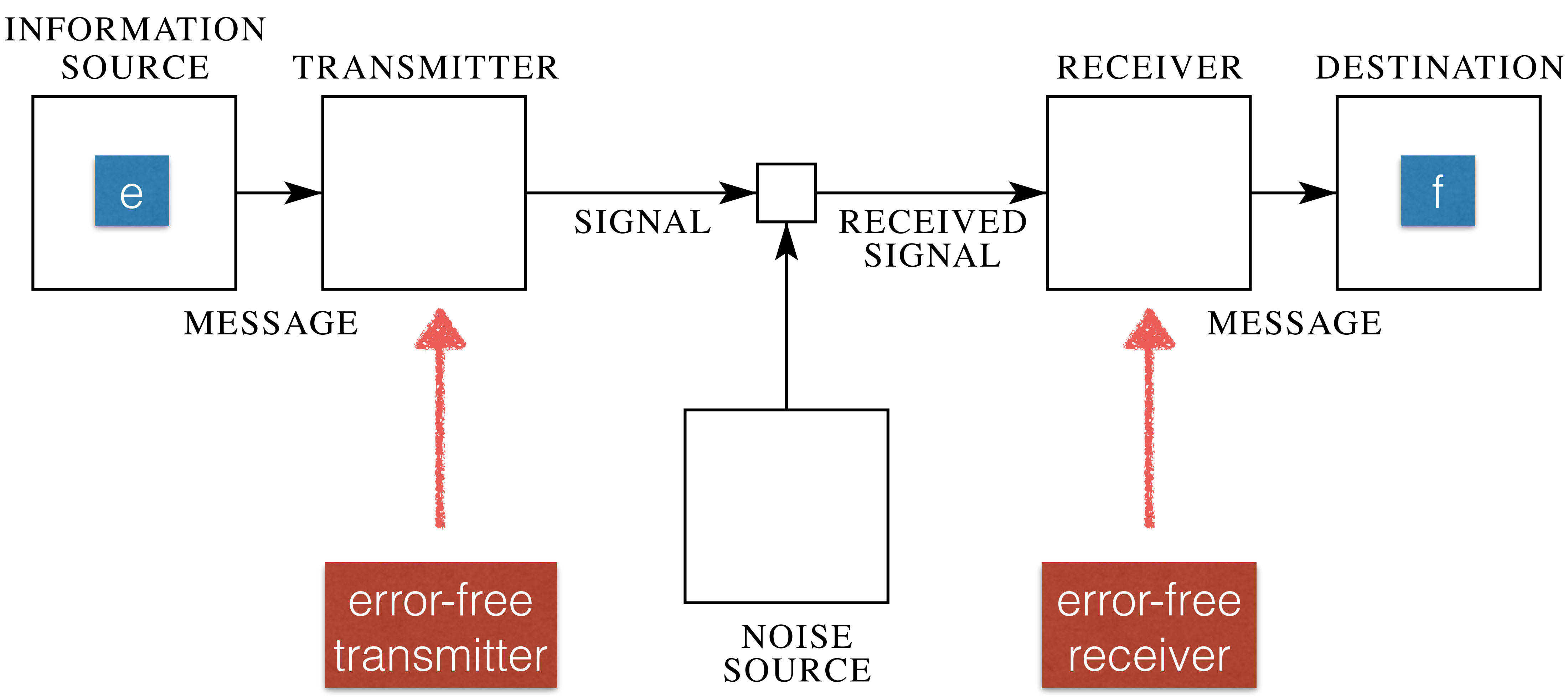

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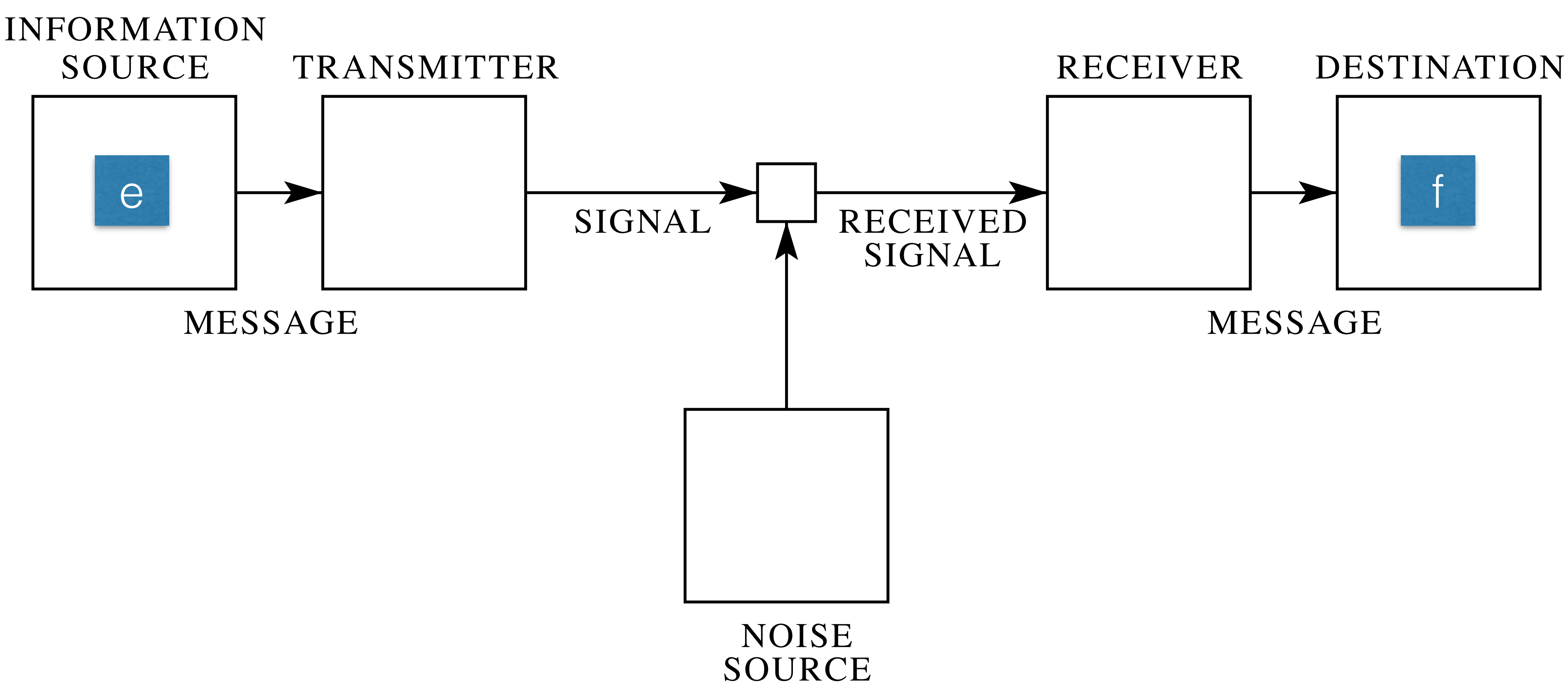

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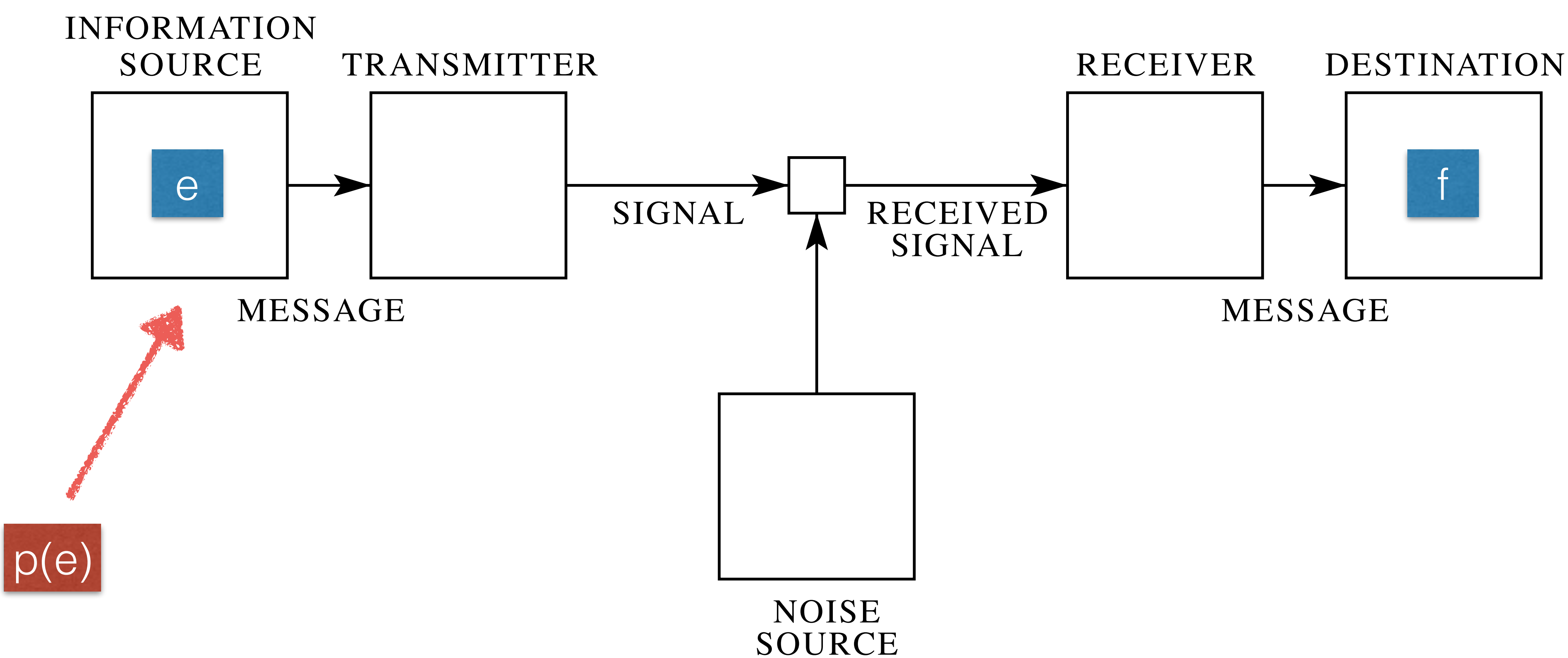

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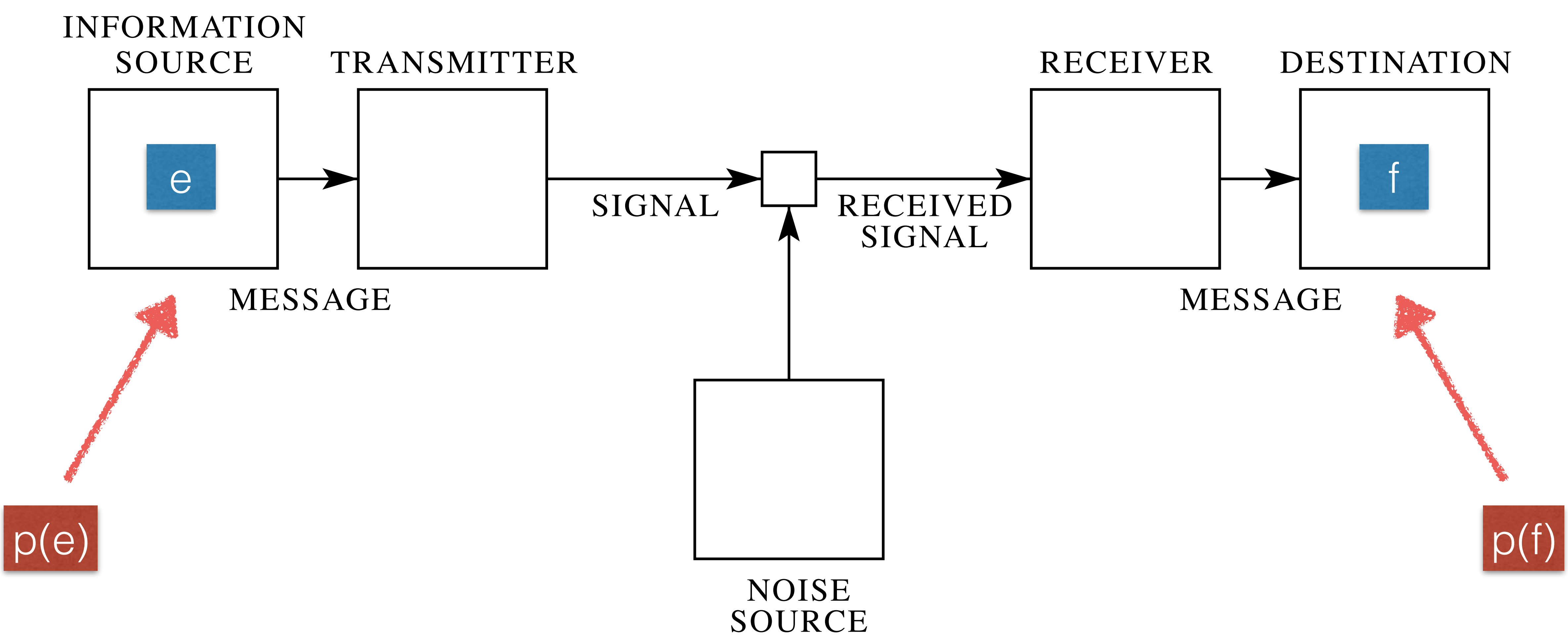

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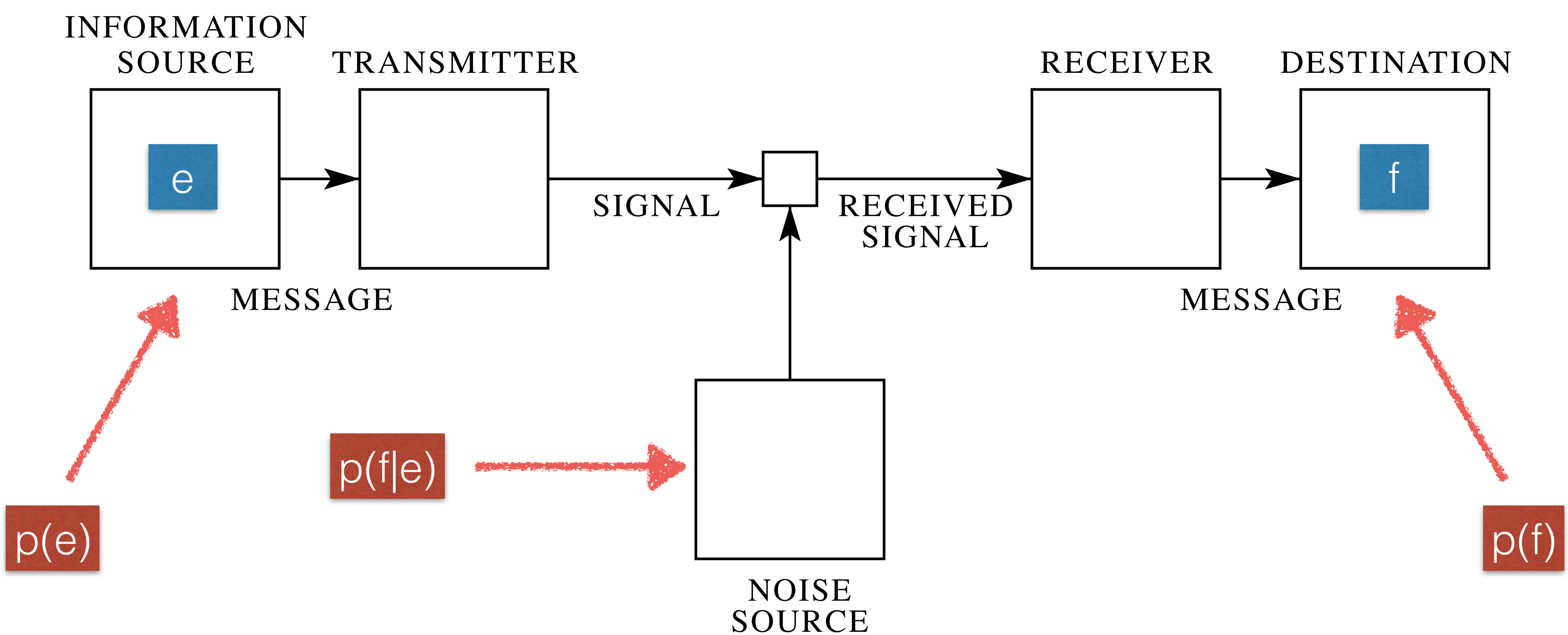

Bayes to the rescue

- *An Essay towards solving a Problem in the Doctrine of Chances*. Philosophical Transactions of the Royal Society of London. 1763
- Bayes's Law:

$$p(\mathbf{e}|\mathbf{f}) = \frac{p(\mathbf{f}|\mathbf{e})\, p(\mathbf{e})}{p(\mathbf{f})}$$

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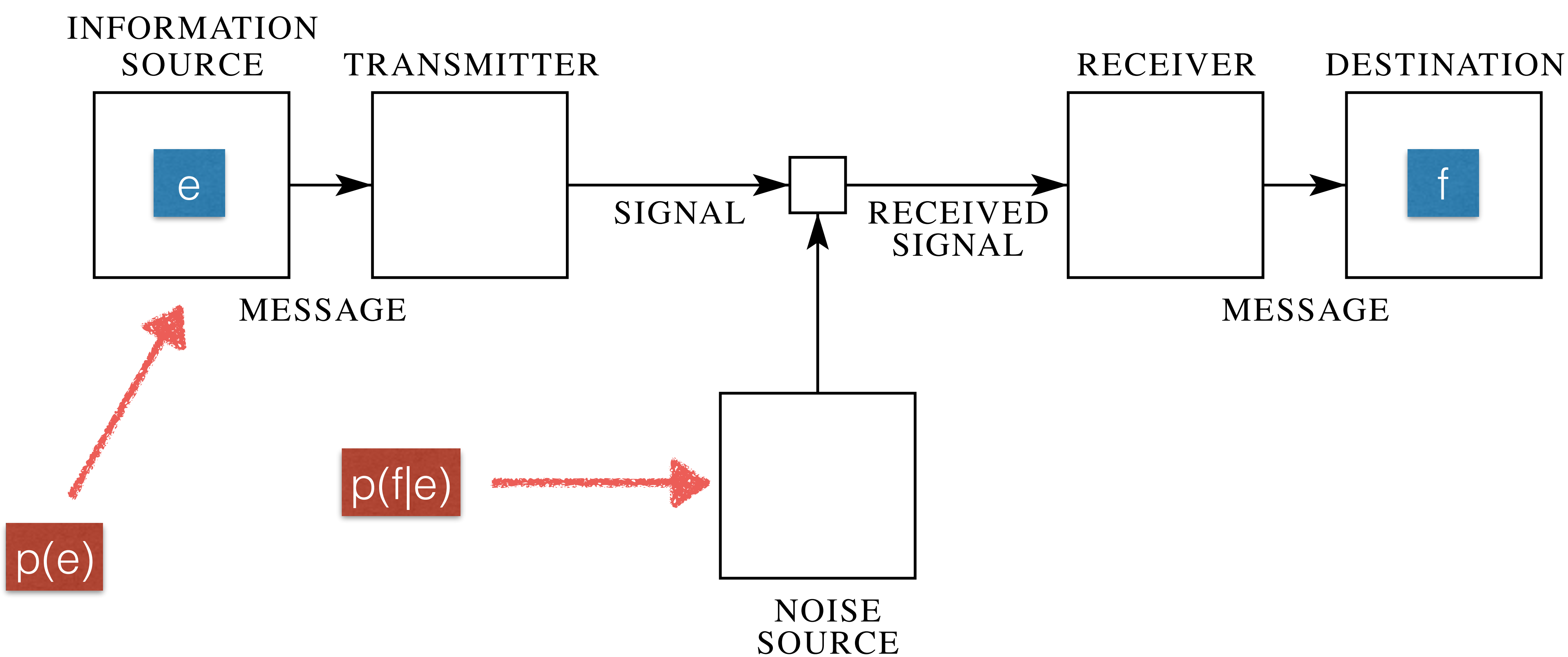

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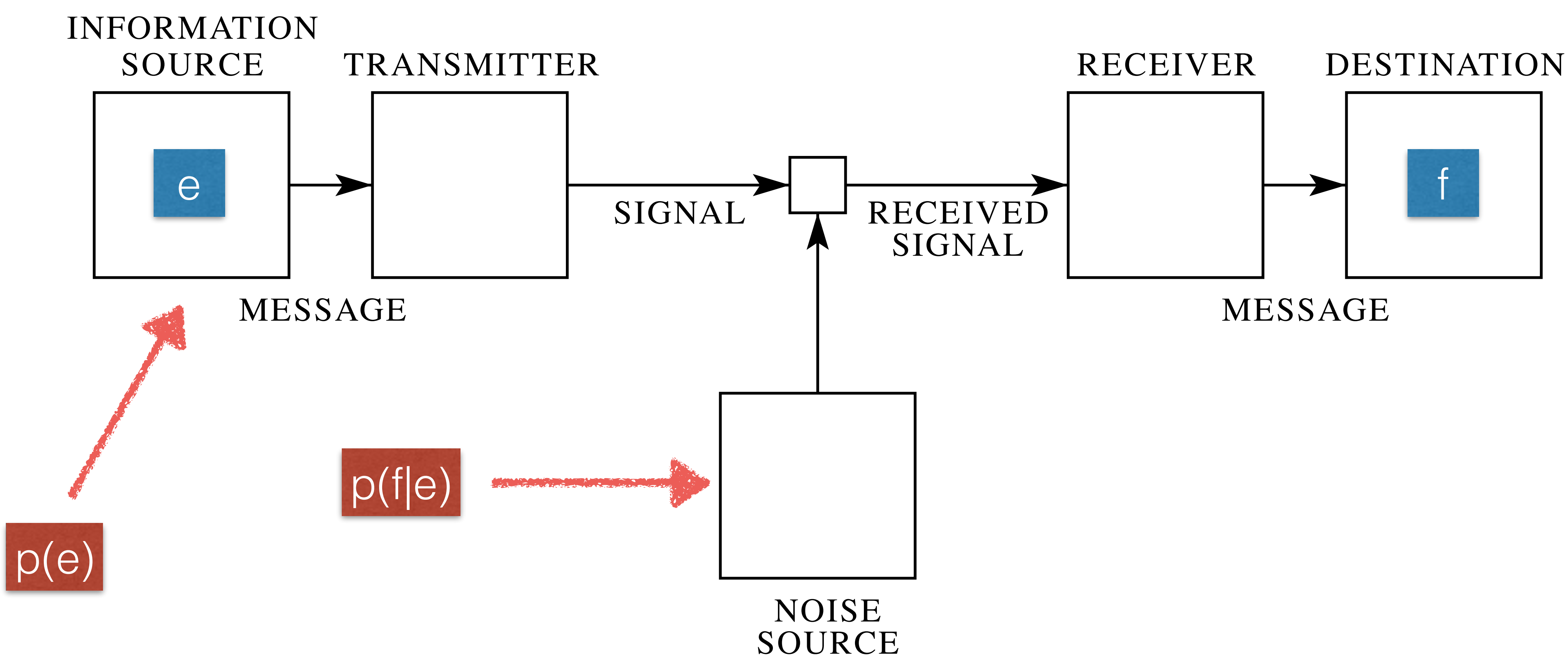

References (Primary)

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- *A Mathematical Theory of Communication*. Claude Shannon. The Bell Systems Technical Journal. October 1948.

References (Secondary)

- *The Mathematics of Communication*. Warren Weaver. Scientific American. July 1949.
- *Mathematical Theory of Claude Shannon*. Eugene Chiu, Jocelyn Lin, Brok Mcferron, Noshirwan Petigara, Satwiksai Seshasai. 2001.

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